



Comparative Study of Parametric Vs. Non-Parametric Hypothesis Testing

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Abstract – Data from many fields rely on statistical analysis to help them make sense of it. There are two main schools of thought when it comes to statistical analysis: parametric and non-parametric. Although they share the goal of deducing conclusions from data, their assumptions and guiding principles are different. To help readers choose the right approach for any given situation, this article compares and contrasts the two approaches, including their advantages and disadvantages. The Parametric technique, which is also utilized in Machine Learning, is based on the assumption that a probability model may be determined using a set of defined parameters. When using a parametric technique, one must either have previous knowledge that the population is normally distributed or be able to quickly estimate it with a Normal Distribution, a feat made feasible by the Central Limit Theorem. One must be able to make assumptions about the data's population distribution in order to choose between non-parametric and parametric hypothesis testing. Non-Parametric analyses are more versatile (distribution-free), but they may not be as strong as parametric tests, which often need tighter assumptions.

Keywords – Parametric technique; Statistical Methods; Machine Learning; Non-Parametric; Hypothesis Testing.

1. BACKGROUND

In almost every instance, the results obtained from using parametric analysis on actual survey data are identical to those obtained from using non-parametric analyses. Three primary purposes of statistical methods are as follows [1], [2]: (1) generating hypotheses and designing experiments to test them; (2) synthesizing information to provide straightforward, clear, and significant understanding; and (3) analyzing quantitative data to draw valid conclusions about the phenomena observed. It is common practice to use either parametric or non-parametric approaches for these three primary purposes. The standard

deviation and mean of a normal or Gaussian distribution form the basis of parametric approaches. Within two standard deviations of the mean, 95% of the findings fall into a symmetric distribution around the mean [3], [4]. Neither these parametric distributions of probability nor any presumptions on the data's distribution of probability form the basis of nonparametric statistics. In order to demonstrate that intervals or differences are equivalent, parametric statistical techniques are used to continuous, interval data. Ordinal data, which includes the choice of "larger" or "smaller," i.e., the ranking of data, is treated using non-parametric approaches. This includes Likert scale data [6].

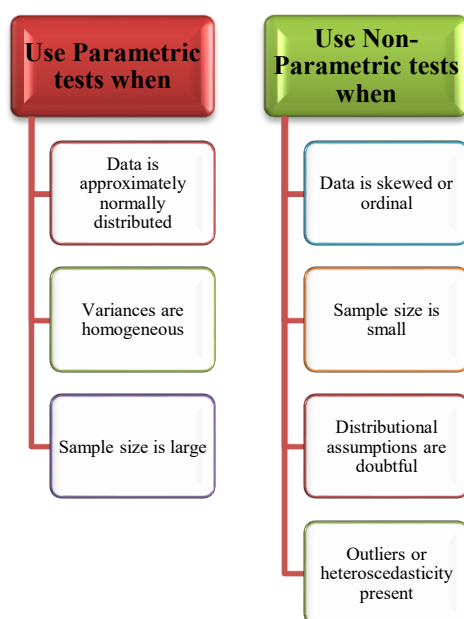


Fig. 1: Parametric and Non-Parametric Test Uses

A scalar significance thresholding function may be adaptively created in multiple simultaneous hypothesis testing (MSHT) by exchanging information across numerous tests that are run at the same time. With this modified statistic, MSHT outperforms tests that rely on individual statistics alone in terms of detection power. Storey (2007) put forth a theoretical framework known as the optimum discovery method to systematically find the ideal thresholding function that optimizes the finding power in MSHT. In addition, he suggested a method for empirically estimating the ODP thresholding function for a parametric MSHT, which assumes that the null and alternative likelihood functions are parametric. On occasion, the non-parametric approach to arbitrary test statistics used in empirical Bayesian testing (Efron et al., 2001) has shown results that are similar to those of the ODP. Due to methodological differences (frequentist vs. Bayesian), the nature of the link between these two MSHT frameworks remains unclear, despite their apparent closeness. We provide a novel idea of an optimum adequate statistic that bridges the gap between ODP and empirical Bayesian structures and demonstrate that, under some circumstances, the local false discovery rate derived from empirical Bayes may serve as an ideal thresholding function. We provide all the necessary assumptions to get optimum thresholding functions and prove that an optimal function obtained for a parametric MSHT issue is also optimal for a larger class of non-or semi-parametric MSHT problems [6].

Thus, our work [7] provides a reference for building effective thresholding functions for broad MSHT situations. The self-consistent approach may be used to generate the non-parametric maximum probability estimate of the distribution function using doubly censored data (Turnbull, 1974). A restriction on the NPMLE is included into the self-consistent algorithm as an extension. After that, we demonstrate how to use the empirical likelihood proportion to build confidence intervals and evaluate NPMLE-based hypotheses. In the end, we provide numerical comparisons of how well the aforementioned technique and another one that uses impact functions [8] fare. This study compares two popular approaches to estimating option implied Risk-Neutral Densities using statistical methodologies. Two of these methods are the volatility function methodology and the mixture of lognormals. Unlike the latter, which is a non-parametric technique, the former is a parametric method. The Mexican peso-US dollar exchange rate OTC European-style options are the source of the RNDs. If you need to conduct an assessment outside of a sample, the non-parametric approach is the way to go. There was a statistically significant difference between the competing methods with respect to the suggested mean, median, and mode. Due of its

greater accuracy and its applicability if there is a relatively limited cross-section of the option exercise price range, the VFT is preferred over the MXL. Financial investors and policymakers may utilize the data to guide their decisions by analyzing market views of the financial asset's predicted future variability via options [9].

In this study, we look at how well various parametric and non-parametric subclasses of univariate models predict the returns of the South African stock market. The non-parametric model is created through the conditionally heteroskedastic non-linear autoregressive approach, which takes conditional heteroskedasticity in stock returns data into consideration. The parametric model is then generated by the generalized autoregressive conditional heteroskedastic in mean approach. The paper's findings highlight the significance of a distribution-free approach for forecasting stock returns in South Africa, since the NAR, a non-parametric model, outperforms the GARCH-M approach in short-term forecasting horizon. The number ten. One of the most basic challenges in signal or array processing is counting the signals buried in noise. In this work, we assume nothing about the array manifold and concentrate on the non-parametric case. We begin with a comprehensive statistical study of the issue, which includes a look at the best detection test under specific conditions when one exists and the signal intensity needed for high probability detection. Secondly, we provide a novel approach for detecting the total number of sources via a series of hypothesis tests by integrating this analysis using recent findings from random matrix theory. By conducting a theoretical analysis of the suggested algorithm's consistency and detection performance, we demonstrate that it outperforms the conventional MDL-based estimator. A number of computational experiments back up our theoretical findings [11].

2. PARAMETRIC TESTS

While non-parametric tests do not rely on assumptions about the population or data sources, parametric tests do. Mean, standard deviation, deviation, and other parameters make up parametric statistics. As a result, it extrapolates distribution parameters from the observed data. It is common practice to presume that data follows a normal distribution having parameters that are not known. A parametric test is one that uses a predetermined set of parameters to predict the likelihood that a given sample will represent a population that normally distributes its data.

2.1 Parametric Test Types

Z Test

The purpose of a one-sample z-test is for comparing the sample mean with a presumptive

value, often the population mean. Important prerequisites for the test include knowing the population's standard deviation and having a sample size greater than 30.

One sample

An alternative parametric test called the one-sample t-test may be used if neither of the aforementioned conditions can be satisfied. Here, you may use this test if you know the sample's standard deviation and the sample size is more than or equal to 15. It is expected that the sample distribution will be close to normal in this case. The One-Sample T-test the process of comparing the average of a population with a sample.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Where,

- \bar{x} is the sample mean
- s is the sample standard deviation
- n is the sample size
- μ is the population mean

Paired (dependent) t test

If you have data gathered from an identical subject prior to and subsequent to an event, for example, you may compare the means of the two groups using a paired t-test. Assumptions include that groups are autonomous, that pre- and post-test scores are from the identical subjects, and that intergroup differences follow a normal distribution.

- Raw Value Dependence: By using the mean (\bar{X}) and standard deviation (s), the test statistic is directly computed from the raw data values (X_i).
- Formula Example (T-statistic):

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

Since the squared variance from the central point is used by the mean and standard deviation, squaring differences causes bigger values to have a disproportionate numerical effect, this is particularly sensitive to the precise size of every observation.

Two Sampled (Independent) t-Test

It is possible to employ a two-sampled t-test when there are two independent samples if there is a statistically significant difference between their

means. Assumptions include a normally distributed distribution for the data, continuous values, an identical variance for the two samples, and their independence from one another.

When comparing the means of two separate samples, the two-sample t-test is used.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$$

Where,

- S_1 is the sample-1 standard deviation
- S_2 is the sample-2 standard deviation
- n is the sample size
- \bar{x}_1 is the sample mean of the first group
- \bar{x}_2 is the sample mean of the second group

2.2 One-way Analysis of Variance

When comparing more than two groups, one-way ANOVA is an extension of two-sampled t-tests. To address the question directly, yes, analysis of variance (ANOVA) is a parametric test. The normality of the population distribution, homogeneity of variance, and independence of the groups are all prerequisites for analysis of variance (ANOVA), which examines the variance of the groups. If you take a sample from a population and make assumptions about its parameters, you are doing a parametric test.

1. Normality: The test statistic's sample distribution or the population's data should roughly follow a normal distribution, often known as a bell curve.
2. Data on intervals or ratios: This data must be continuous, with significant distances between values, and measured on a proportional or interval scale.
3. Groups that are compared should have almost identical variances; this is known as homogeneity of variance.
4. Central Tendency: Usually, they examine theories connected to the average of the population (μ).

When the data satisfies the strict requirements, parametric tests are used to enable stronger and more accurate conclusions on the population.

Table 1: Parametric Test and Hypothesis Tested

Goal	Parametric Test	Hypothesis Tested
Compare three or more groups	One-Way Analysis of Variance (ANOVA)	Equality of means ($\mu_1=\mu_2=\mu_3...$)
Assess linear association	Pearson's Correlation Coefficient (r)	Linear relationship between two continuous variables
Compare two independent groups	Independent Samples T-Test	Difference in means ($\mu_1=\mu_2$)
Compare two related (paired) groups	Paired Samples T-Test	Difference in means ($\mu_D=0$)

3. NON-PARAMETRIC TESTS

In order to be considered non-parametric, statistics must be either distribution free or have a predetermined distribution; nonparametric tests are those that do not take parameters into account and do not assume anything based on a sample population. Since they do not depend on data associated with any specific parametric set of probability distributions, non-parametric tests are often called distribution free tests. As compared to the parametric tests, it is more grounded in actuality. Studies in which an overall change in input has a little or no impact on output are often better suited to non-parametric statistics. The outcomes are expected to remain unchanged regardless of any adjustments made to the numerical data. When conducting statistical analyses, non-parametric approaches avoid making any presumptions on the distribution of the population under study. These approaches are typically called "distribution-free" techniques as they do not presume any particular distribution form.

No assumptions about the parameters of the provided or studied population are necessary according to the parametric method's central tenet. Actually, the population is irrelevant to the procedures. There isn't a predetermined set of parameters or any form of distribution (normal, otherwise) to work with here. This is why distribution-free approaches and nonparametric methods have the same name. In recent times, non-parametric approaches have become more prominent. One explanation for this is:

- For one, we don't have to be polite while utilizing parametric approaches.
- Secondly, we don't have to assume more and more about the population we're dealing with, which is a huge relief.
- The complexity level of the majority of the current nonparametric approaches is minimal, making them both straightforward to use and comprehend.

Types of Non-Parametric Tests

1. Wilcoxon Rank Sum Test

Rank sum test, signed rank test, and Wilcoxon test are acronyms for the same thing. This test uses two matched groups and is non-parametric in nature. The test is useful for determining and analyzing the differences between every pair. To determine whether there is a difference in the population mean rank between two samples, the Wilcoxon test is used. This test may be applied to matched samples, related samples, or numerous measurements on a single sample.

2. Mann- Whitney U Test

Various names for the Mann-Whitney U test include Wilcoxon rank sum test, Mann-Whitney-

Wilcoxon, and Wilcoxon-Mann-Whitney. The null hypothesis test is a non-parametric statistical method. A randomly chosen subset of one sample may be more valuable than the other, or it could be less valuable. The chances of either outcome are equal. If the dependent variable is ordinal or continuous, then the Mann-Whitney U test is the way to go for making comparisons between the two groups. However, a normal distribution shouldn't be applied to these variables.

3. Kruskal Wallis Test

The Kruskal-Wallis H test, also known as a one-way Anova on ranks, compares the means of two or more groups using an independent variable. Analysis of variance is the enlarged form of the term ANOVA. The test's primary objective is to determine whether or not the sample represents the same population.

4. Friedman Test

Both the Kruskal Wallis and the Friedman tests are comparable. Compared to the ANOVA test, it is not as invasive. The repeated-measures premise of the Friedman test is the only distinction between it and the ANOVA test. While the dependent variable is determined in ordinal, the Friedman test is used to generate differences between the two groups. Two subtests, the Friedman 1 and the Friedman 2 tests, make up the Friedman analysis. Because it works with whole block designs as well, this test is often called a special instance of the Durbin test.

5. Chi-square test

If you want to be sure that two categorical variables are independent, you may use the chi-square test, which is also called the goodness of fit test. The variables are discrete and categorical, the groups being tested are not necessarily equal (parametric tests assume that the groups are about equal), and there is no homoscedasticity in the data. These are just a few of the many reasons for this. Assumptions on the shape of the distribution of the population are not made by non-parametric tests. Instead, then using the raw data numbers, they often use the data's rankings or signals.

1. Distribution-Free: Does not assume, or only makes limited assumptions about, the distribution of the population.
2. Type of Data: Apt for Ordinal, Non-Normal, or Nominal Interval/Ratio Data.
3. Central Tendency: Usually, they check whether the population's median or distribution's shape and location are relevant hypotheses.
4. Robustness: They can withstand instances of non-normal data and outliers with more ease.

When parametric test assumptions are not satisfied, such as in cases of very skewed data or small sample sizes, non-parametric tests should be used.

Table 2: Non-Parametric Test (Counterpart to Parametric) and Hypothesis Tested

Objective	Non-Parametric Test (Counterpart to Parametric)	Hypothesis Tested
Compare three or more groups	Kruskal-Wallis H Test	Distributions or medians of a population that are equal
Assess monotonic association	Spearman's Rank Correlation (ρ)	The direction of change (monotonic connection) between two variables
Compare two independent groups	Mann-Whitney U Test (or Wilcoxon Rank-Sum)	Distributions or medians of a population that are equal
Compare two related (paired) groups	Wilcoxon Signed-Rank Test	Variation in the middle values of two sets of results

5. Comparative Study of Hypothesis Testing

Power and flexibility/assumptions are two factors to consider when deciding whether to utilize a parametric or non-parametric test.

Table 3: Rationale and Recommended Test Type

Setting	Rationale	Recommended Test Type
Ordinal Data (e.g., Likert scale)	Tests based on ranks are more suitable since the data scale is not really continuous (interval).	Non-Parametric
Data with Extreme Outliers	By using rankings, non-parametric tests reduce the impact of outlying quantities.	Non-Parametric
Large Sample, Normal Data	Enhancement of statistical power and accuracy in estimating means.	Parametric
Small Sample, Skewed Data	Non-Parametric is resilient; violates the notion of normalcy; the mean is not as representative.	Non-Parametric

The mean and variance of a population form the basis of parametric tests, which presume that the data follows a specific pattern (often the Normal distribution). Unlike parametric tests, which

assume a normal distribution for the data, non-parametric tests do not and instead rely on rank or median data.

Table 4: Assumptions

Feature	Parametric Tests	Non-Parametric Tests
Sample size	Survive while dealing with big datasets (Central Limit Theorem)	Helpful for dealing with biased or tiny samples
Outliers	Exposed to extreme cases	Stronger against extreme cases
Distribution	Need certain presumptions (such as normality and equal variance)	We do not assume a strict distribution.
Measurement scale	Data presented as intervals or ratios (metrical)	Scale: ordinal, ratio, interval, or nominal

Table 5: Common Examples

Test Type	Parametric Tests	Non-Parametric Alternatives
Correlation	Pearson correlation	Spearman's rank correlation, Kendall's tau
Regression	Linear regression	Rank regression, Theil-Sen estimator
Goodness of fit	χ^2 (Chi-square) under certain assumptions	Kolmogorov-Smirnov test, Runs test
Compare means (2 groups)	Independent t-test, Paired t-test	Mann-Whitney U test, Wilcoxon signed-rank test
Compare means (3+ groups)	One-way ANOVA, Repeated measures ANOVA	Kruskal-Wallis test, Friedman test

Table 6: Advantages & Disadvantages

Feature	Parametric Tests	Non-Parametric Tests
Flexibility	Interval and ratio data only	Able to process data that is ordinal, skewed, nor categorical
Robustness	Inadequate resistance to violation of assumptions	Extremely secure against breaches of assumptions
Power	In most cases, more potent when certain conditions are satisfied	Weak, but protected against violations
Interpretation	Connected findings to averages and standard deviations (more easily understood)	Results derived from medians and rankings (less user-friendly)

5. Numerical Illustration (Example)

What if we were to compare the test results of two groups of students?

- **Group A (n=10):** 75, 78, 80, 82, 85, 90, 92, 95, 98, 100
- **Group B (n=10):** 60, 62, 65, 68, 70, 72, 75, 77, 80, 82
- **Parametric test (t-test):** assumes normality → shows significant difference ($p < 0.05$).

- **Non-parametric test (Mann-Whitney U):** uses ranks → displays a notable disparity as well, however it does not presume normalcy.

Interpretation: While both tests find the same thing, the t-test might be deceptive with skewed distributions, while the Mann-Whitney U test would still be valid.

Feature	Non-Parametric Tests (E.g., Mann-Whitney U, Kruskal-Wallis, Spearman's ρ)	Parametric Tests (E.g., T-Test, ANOVA, Pearson's r)
Statistical Power	When all the conditions are satisfied, the power is lower than its parametric counterpart.	Greater strength when all conditions are satisfied, particularly normalcy. More probable to notice an actual impact.
Sensitivity to Outliers	By using rankings or medians, they become less affected by outliers.	Extremely susceptible to skewed data and outliers, which have the potential to significantly alter the mean.
Sample Size	Used most often with smaller samples when establishing normalcy is difficult or impossible.	This method is better suited for bigger samples ($n \geq 30$) as it helps meet the normality condition for the sample mean using the Central Limit Theorem.
Output / Interpretation	Might be trickier; hypotheses often center on differences in medians or rankings rather than means.	Simple; gives confidence ranges and estimates of population metrics like mean difference.
Core Assumption	Does not presume anything about the distribution of the population (distribution-free).	The Normal Distribution is the most common distribution used to pull data from populations.
Measure of Central Tendency	Ranks data or uses the Median.	Takes into account the Mean (\bar{v}).
Data Type Required	Data that does not conform to parametric assumptions, whether it be nominal, ordinal, or continuous...	Time interval or ratio (ongoing information).

6. CONCLUSION

There are benefits and drawbacks to both parametric and non-parametric approaches. Picking the best approach for a given analysis requires an understanding of these distinctions. Researchers are able to get meaningful conclusions from their information when they use the correct procedure, which guarantees accurate and reliable inferences. Parametric and non-parametric approaches alike will be pivotal in the future of statistical analysis, which will include a wide range of disciplines. For non-parametric approaches, the proper null

hypothesis form is subjective when comparing data from various distributions. The equality of means null hypothesis is appropriate for parametric testing. The results demonstrate that the suggested nonparametric test statistics are location-sensitive and shape-sensitive, much as standard non-parametric tests. The suggested non-parametric tests are inappropriate for evaluating variations in location if the sample distributions are not assumed to be similar.

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