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Eigenfunctions and Eigenvalues for Fourth-Order Inhomogeneous ODE with Boundary Conditions

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Abstract **Original Research Article**

The importance of teaching linear differential equations is demonstrated by the fact that all scientific and technical phenomena are differential equations that are expressed and described in the mathematical sciences. Modern comparative mathematics, which contains several papers pertaining to population growth, engineering, economics, natural science, and technological issues, is fundamentally based on differential equations. A wide range of physics topics are also covered, such as heat, mechanics, atoms, electronics, magnetism, light, and waves. This article examines the theory of eigenvalues and eigenfunctions with boundary conditions in fourth-order linear inhomogeneous differential equations with coefficients of constant and boundary conditions. A boundary value problem in differential equations is characterized as a boundary value problem with a number of extra constraints. Applying the given constraints to the answer of the boundary value problem allows one to solve the differential equation with the given constraints. The boundary value matter's conditions are actually met by this. Initial value problems and differential equation problems with boundary conditions are comparable. For a system of equations, the initial value problem is the condition that has the value of the independent variable. An independent variable in the equation is specified in a boundary value problem with conditions on the boundaries, and the initial value is the data value that matches the minimum or maximum input, internal, or output value specified. This work aims to address the problem of eigenvalues and eigenfunctions with boundary conditions. When the fourth order differential equations border of the boundary values in the solution are reached D_1, D_2, D_3, and D_4 are determined, the boundary problem is the inability to obtain the constants. We resolve this matter by taking into account the specified parameters for the actual Green's function. A set of linear differential equations has a solution for any real function. So, we consider eigenfunctions and eigenvalues problem.

Keywords: Boundary Conditions, Eigenfunctions and Eigenvalues, Green's Function.

1. INTRODUCTION

Differential equations are one of the most intriguing and frequently used mathematical topics, and many academics have expressed interest in it. Physics is one of the many disciplines that uses differential equations. They are especially useful for examining the motion of free vibrations, electric circuits, springs, and attached weights. In mathematics, the boundary value issue of a differential equation with an extra set of restrictions is known as the boundary condition problem. The solution that meets the given requirements also satisfies [1-3]. Numerous branches of physics may encounter boundary value issues since all equations have a differential body. They frequently occur in relation to wave equation problems, like determining normal modes. Between boundary value issues and Green's functions of differential equations, Sturm-Liouville problems make up another significant class. These issues are investigated through the use of eigenfunctions [3, 4]. The formulation and understanding of the problem of solving fourth-order inhomogeneous linear differential equations with boundary conditions and determining Green's function are covered in this article. In the space $L_2(0,1)$ we analyse the boundary problem of an inhomogeneous linear differential equation of the fourth with boundary conditions, and on that starting function is obtained by a linear differential equation of the third order with constant coefficients [2].

Suppose we consider the fourth order differential equation as follows:

$$y^{(4)}(x,\lambda)+P_3(x)y^{(3)}(x,\lambda)+P_2(x)y''(x,\lambda)+P_1(x)y'(x,\lambda)+P_0(x)y(x,\lambda)=f(x) \qquad (1)$$

where $P_0(x,\lambda), P_1(x,\lambda), P_2(x,\lambda)$ and $P_4(x,\lambda)$ many functions on intervals $[0,1]$, the number 4 expresses the order of the differential equation.

With the following boundary conditions, we examine the known characteristics of these functions in this section:

$$U_j(y)=\alpha_j y^{(\gamma_j)}(0)+\beta_j y^{(\gamma_j)}(1)=0, \quad j=1,2,3,4 \qquad (2)$$

It should be known that

$$\gamma_1=0,\ \gamma_2=1,\ \gamma_3=2,\ \gamma_3=3$$

Is recalled

The following queries can be derived from the problem statement above:

Research Questions:

1. How can we solve the inhomogeneous linear differential equation of the fourth order $y^{(4)}(x)+P_2(x)y''(x)=f(x)$ with boundary conditions $U_j(y)=y^{(\gamma_j)}(0)+y^{(\gamma_j)}(1)=0, j=1,2,3,4$?
2. How can we solve the private and general solution of the inhomogeneous fourth order linear differential equation $y^{(4)}(x)+P_2(x)y''(x)+P_1(x)y'(x)+P_0(x)y(x)=f(x)$ if it is with $P_2(x)\neq 0,\ P_1(x)\neq 0,\ P_0(x)\neq 0$ and given boundary conditions $U_j(y)=y^{(\gamma_j)}(0)+y^{(\gamma_j)}(1)=0, \quad j=1,2,3,4$?

The bases for the opinion framework are fourth order differential equations with boundary conditions, Green's function, eigenfunctions, and eigenvalues. From these, a general solution and a private solution on differential equations, boundary conditions, and the Wronski determinant may be obtained.

In order to derive a general solution and a private solution based on boundary conditions, the Wronski determinant, and differential equations, the opinion framework is based on fourth order differential equations with boundary conditions, Green's function, eigenfunctions and eigenvalues.

The paper uses well-tested methods of Greens function analysis and the theory of function variables, results of the spectral theory of unbounded operators, and the methodology of the theory of ODE on Eigenfunctions and Eigenvalues problems and boundary problems.

This research is divided into six basic parts: introduction, review of scientific works, basic concepts, research findings, controversy and conclusion.

2. LITERATURE REVIEW

Differential equations and the relationship between function derivatives and transformations have been known for almost three centuries. Therefore, the discovery of the derivative by the English scientist Isaac Newton between 1642 and 1772 unavoidably marks the beginning of its history. Furthermore, between the years 1646–1716, German scientist Gottfried Leibniz researched differential equations, specifically first-order differential equations. Jacob first proposed the Bernoulli differential equation in 1674, but he did not validate it until Euler did so in 1705. The boundary problem with the initial boundary in linear differential equations was proposed by Sturm- Liouville, Jacques Francois Sturm and Joseph Liouville are the names of the classical Sturm- Liouville theory and its applications, which was put out between (1855-1803) and in (1809-1882) in the second order, the theory of linear differential equations was developed in 1969. The Green's function is a method for solving differential equations with boundary conditions that was described by Russian physicist Naimark in his book Linear Differential Functions. The boundary conditions are strictly regular and defined, according to the theories of Mikhailov and Kesselman [4], since there is a positive integer like this, the asymptotic equation's eigenvalues are straightforward and distinct. δ, for both eigenvalues of the function [3], which are farther apart from one another by a larger distance δ. It is also concluded from the works [1-15] that in the space, the system of eigenfunctions and associated functions forms a basis Res. Many pure mathematicians have been working on finding Green's function for linear differential equations in the last few years. One of the scientists is Kanguzhin, a scientist from Kazakhstan who published an article titled "considering Green's function for second-order linear differential equations" in 2019 [4-6].

3. ELEMENTARY BASIC

The general form of inhomogeneous linear differential equations, taking differential equations into account, can be written as follows:

$$L(y) = \lambda y(x) + f(x) \tag{3}$$

When examining the system of high-order linear differential equations of the general solution of equations, the following initial function might be taken into consideration (1) and (2):

$$y(x,\lambda) = y_H(x,\lambda) + D_1\varphi_1(x,\lambda) + D_2\varphi_2(x,\lambda) + D_3\varphi_3(x,\lambda) + D_4\varphi_4(x,\lambda) \tag{4}$$

Where

$$y_H(x,\lambda) = \int_0^x g(x,t) f(t,\lambda)\,dt$$

$y_H(x,\lambda)$ is the inhomogeneous solution of the above equation and $\varphi_1(x,\lambda), \varphi_2(x,\lambda),\ \varphi_3(x,\lambda), \varphi_4(x,\lambda)$ The main system of solving the equation with homogeneous conditions $L(\varphi_1)=0,\ L(\varphi_2)=0,\ L(\varphi_3)=0,\quad L(\varphi_4)=0$ is one of the inhomogeneous boundary conditions $\varphi_j^{(k-1)}(0) = \delta_{kj}$ function $g(x,t)$ it is determined by the following formula, which can be called Green's function [9,12].

$$g(x,t) = \frac{P(x,t)}{W(t)}$$

where $\delta_{kj} = \begin{cases} 1, & k = j \\ 0, & k \neq j \end{cases}$ and $W(t,\lambda)$ Wronski determinant

$$W(t_1,t_2,t_3,t_4) = \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_2,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1'''(t_1,\lambda) & y_2'''(t_2,\lambda) & y_3'''(t_3,\lambda) & y_4'''(t_4,\lambda) \end{vmatrix}$$

and it should be known that $P(x,t)$ is equal to:

$$P(x,t) = \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1(x_1,\lambda) & y_2(x_2,\lambda) & y_3(x_3,\lambda) & y_4(x_4,\lambda) \end{vmatrix}.$$

So, you should know that $g(x,t) = P(x,t)$ so $g(x,t)$ can be defined from the following formula.

$$g(x,t) = \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1(x_1,\lambda) & y_2(x_2,\lambda) & y_3(x_3,\lambda) & y_4(x_4,\lambda) \end{vmatrix}.$$

From here we can propose a specific homogeneous solution as follows:

$$y_H(x,\lambda)=\int_0^x \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1(x_1,\lambda) & y_2(x_2,\lambda) & y_3(x_3,\lambda) & y_4(x_4,\lambda) \end{vmatrix} f(t,\lambda)dt.$$

The inhomogeneous solution function $y_H(x,\lambda)$ is equation (1), (2) and for its correctness, we search the first, second, third and fourth order derivatives and establish the proposed fourth order equation (1).

$$y_H'(x,\lambda)=\int_0^x \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1'(x_1,\lambda) & y_2'(x_2,\lambda) & y_3'(x_3,\lambda) & y_4'(x_4,\lambda) \end{vmatrix} f(t,\lambda)dt$$

now we take the second derivative

$$y_H''(x,\lambda)=\int_0^x \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1''(x_1,\lambda) & y_2''(x_2,\lambda) & y_3''(x_3,\lambda) & y_4''(x_4,\lambda) \end{vmatrix} f(t,\lambda)dt$$

then in the same way, we get the derivative of the third order

$$y_H'''(x,\lambda)=\int_0^x \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1'''(x_1,\lambda) & y_2'''(x_2,\lambda) & y_3'''(x_3,\lambda) & y_4'''(x_4,\lambda) \end{vmatrix} f(t,\lambda)dt$$

eventually in the same way, we get the derivative of the fourth order

$$y_H^{[4]}(x,\lambda)=\int_0^x \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1^{[4]}(x_1,\lambda) & y_2^{[4]}(x_2,\lambda) & y_3^{[4]}(x_3,\lambda) & y_4^{[4]}(x_4,\lambda) \end{vmatrix} f(t,\lambda)dt+f(x)$$

so, for the correctness of the received function, we must establish and check the price of the function and its derivatives of different degrees in shortly equation (1).

$$L(y)=y^{[4]}(x,\lambda)+P_2(x,\lambda)y''(x,\lambda)+P_0(x,\lambda)y(x,\lambda)=f(x)$$

$$L(y)=\int_0^x \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1^{[4]}(x_1,\lambda) & y_1^{[4]}(x_2,\lambda) & y_1^{[4]}(x_3,\lambda) & y_1^{[4]}(x_4,\lambda) \end{vmatrix} f(t,\lambda)dt+f(x)$$

$$+P_2(x,\lambda)\int_0^x \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1''(x_1,\lambda) & y_2''(x_2,\lambda) & y_3''(x_3,\lambda) & y_4''(x_4,\lambda) \end{vmatrix} f(t,\lambda)dt$$

$$+P_0(x,\lambda)\int_0^x \begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1(x_1,\lambda) & y_2(x_2,\lambda) & y_3(x_3,\lambda) & y_4(x_4,\lambda) \end{vmatrix} f(t,\lambda)dt$$

from here we add the determinants together,

$$L(y)=\int_0^x \begin{vmatrix} y_1(t,\lambda) & y_2(t,\lambda) & y_3(t,\lambda) & y_1(t,\lambda) \\ y_1''(t,\lambda) & y_2''(t,\lambda) & y_3''(t,\lambda) & y_4''(t,\lambda) \\ y_1^{[4]}(x,\lambda)+P_2y_1''(x,\lambda)+P_0y_1(x,\lambda) & \dots & \dots & y_4^{[4]}(x,\lambda)+P_2y_4''(x,\lambda)+P_0y_4(x,\lambda) \end{vmatrix} f(t,\lambda)dt+f(x).$$

$$L(y)=f(x).$$

Conditions of homogeneous equation $L(y)=y_1^{[4]}(x,\lambda)+P_2y_1''(x,\lambda)+P_0y_1(x,\lambda)=0$ so that we can solve function of $f(x)$ and as a result we can say that we have obtained the solution of the inhomogeneous part. We have obtained the Green's function for the proposed problem and according to the problem, we have proved that:

$$L(y)=y_1^{[4]}(x,\lambda)+P_2y_1''(x,\lambda)+P_0y_1(x,\lambda)=f(x), \qquad 0<x<1 \tag{5}$$

with boundary conditions

$$U_1(y)=0,\ U_2(y)=0,\ U_3(y)=0,\ U_4(y)=0. \tag{6}$$

The type of boundary conditions that are already defined for us.

$$U_1(y)=\alpha_1 y(0)-\beta_1 y(1)=0$$
$$U_2(y)=\alpha_2 y'(0)-\beta_2 y'(1)=0$$
$$U_3(y)=\alpha_3 y''(0)-\beta_3 y''(1)=0$$
$$U_3(y)=\alpha_4 y''(0)-\beta_4 y''(1)=0$$

It can be said that we can solve the equation and Green's function (5), (6) using Green's functions as follows:

$$y(x,t)=(L_0-\lambda I)^{-1}f=\int_0^1 G_0(x,t,\lambda)f(t,\lambda)dt$$

Where

$$G_0(x,t,\lambda)=\frac{\begin{vmatrix} y_1(x,\lambda) & y_2(x,\lambda) & y_3(x,\lambda) & y_4(x,\lambda) & g(x,t) \\ U_1(y_1) & U_1(y_2) & U_1(y_3) & U_1(y_4) & U_1(g) \\ U_2(y_1) & U_2(y_2) & U_2(y_3) & U_2(y_4) & U_2(g) \\ U_3(y_1) & U_3(y_2) & U_3(y_3) & U_3(y_4) & U_3(g) \\ U_4(y_1) & U_4(y_2) & U_4(y_3) & U_4(y_3) & U_4(g) \end{vmatrix}}{\begin{vmatrix} U_1(y_1) & U_1(y_2) & U_1(y_3) & U_1(y_4) \\ U_2(y_1) & U_2(y_2) & U_2(y_3) & U_2(y_4) \\ U_3(y_1) & U_3(y_2) & U_3(y_3) & U_3(y_4) \\ U_4(y_1) & U_4(y_2) & U_4(y_3) & U_4(y_4) \end{vmatrix}}$$

$G_0(x,t,\lambda)-$ is a Green's function. We assume

$$4\geq\gamma_3\geq\gamma_2\geq\gamma_1\geq\gamma_0\geq 0.$$

4. MAIN RESULTS

$$y(x,t)=(L_0-\lambda I)^{-1}f=\int_0^1 G_0(x,t,\lambda)f(t,\lambda)dt$$

Or

$$y(x,t)=(L_0-\lambda I)^{-1}f=\int_0^1 \frac{\begin{vmatrix} y_1(x,\lambda) & y_2(x,\lambda) & y_3(x,\lambda) & y_4(x,\lambda) & g(x,t) \\ U_1(y_1) & U_1(y_2) & U_1(y_3) & U_1(y_4) & U_1(g) \\ U_2(y_1) & U_2(y_2) & U_2(y_3) & U_2(y_4) & U_2(g) \\ U_3(y_1) & U_3(y_2) & U_3(y_3) & U_3(y_4) & U_3(g) \\ U_4(y_1) & U_4(y_2) & U_4(y_3) & U_4(y_3) & U_4(g) \end{vmatrix}}{\Delta(\lambda)} f(t,\lambda)dt$$

If $x>t$ the function $g(x,t)$ has the following form

$$g(x,t)=\begin{vmatrix} y_1(t_1,\lambda) & y_2(t_2,\lambda) & y_3(t_3,\lambda) & y_4(t_4,\lambda) \\ y_2'(t_1,\lambda) & y_2'(t_2,\lambda) & y_3'(t_3,\lambda) & y_4'(t_4,\lambda) \\ y_1''(t_1,\lambda) & y_2''(t_2,\lambda) & y_3''(t_3,\lambda) & y_4''(t_4,\lambda) \\ y_1(x_1,\lambda) & y_2(x_2,\lambda) & y_3(x_3,\lambda) & y_4(x_4,\lambda) \end{vmatrix}.$$

If $x\leq t$ is then a function $g(x,t)=0$.

$$\Delta(\lambda)=\begin{vmatrix} U_1(y_1) & U_1(y_2) & U_1(y_3) & U_1(y_4) \\ U_2(y_1) & U_2(y_2) & U_2(y_3) & U_2(y_4) \\ U_3(y_1) & U_3(y_2) & U_3(y_3) & U_3(y_4) \\ U_4(y_1) & U_4(y_2) & U_4(y_3) & U_4(y_4) \end{vmatrix}$$

5. DISCUSSION

We infer that the fourth-order inhomogeneous linear differential equation problem we studied is a collection of Green's functions based on the study presented in this article. Every real function can be found in the solution to a set of linear differential equations. In addition to a single definite solution, these equations also have numerous solutions, or mean eigenfunctions and eigenvalues issues with boundary value restrictions. Finding the temperature at each point on an iron rod with one end at absolute zero and the other at the freezing point of water is an example of a boundary value problem, which is used in physics.

For instance, if the eigenfunctions and eigenvalues problems with boundary value constraints depend on both space and time, the value of the problem can be found at a specific point for all times or at a specific time for the entire space. Another example is a linear differential equation with boundary conditions that solves eigenfunctions and eigenvalues problems. The Dirichlet boundary condition is the border condition that determines the function's value using eigenfunctions and eigenvalues. The magnitude’s problem can be calculated at a specific place in space, for instance, by holding one end of an iron rod at zero Celsius degrees.

6. CONCLUSION

Eigenfunctions and eigenvalues under the boundary conditions in theory, we can solve any problem in this system as we have the Green's function for the fourth-order inhomogeneous linear differential equation. We suggested the strategy that was received in order to solve it, and we demonstrated that there are multiple solutions in terms of eigenfunctions and eigenvalues for the fourth-order inhomogeneous linear differential equation with the particular boundary conditions.

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