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An Exact and Simple Solution to "Angle Quintisection" Problem Using Straightedge and Compass

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Abstract **Review Article**

The term "Angle Quinti-section/Penta-section" refers to the process of dividing an angle into five equal sub-angles using only a straightedge and compass within the framework of Euclidean geometry. This problem is derived from the previously published article, "An Exact and Simple Solution to the 'Trisecting an Angle' Problem Using Straightedge and Compass," which appeared in 2025. In ancient Greek mathematics, three classical problems significantly influenced the development of geometry: *Squaring the Circle, Trisecting an Angle, and Doubling the Cube.* Among these, the problem of angle quinti-section (or penta-section) specifically involves constructing an angle that is exactly one-fifth of a given arbitrary angle using only an unmarked straightedge and a compass. This thesis concentrates solely on the quinti-section/penta-section process for arbitrary angles. I present a classical straightedge-and-compass construction that achieves exact penta-section/quinti-section while avoiding the explicit use of π. This approach employs a ruler-based geometric analysis and synthesis. Although quinti-sections/penta-sections for specific angles (such as a right angle) is relatively a feeling idea straightforward, addressing the general case has not been explored within classical constraints. The concept of angle quinti-section/penta-section was formulated in December 2025, as a continuation of my research on the *Angle Trisection,* which was solved precisely and simply and subsequently published in SJPMS [8]. The results of this research provide an exact construction-based solution to the challenge of penta-section/quinti-section process of an angle utilizing only a straightedge and compass in Euclidean geometry. While the solution relies on theorems and corollaries from high school geometry, it remains somewhat complex. Additionally, this study introduces a novel tool, the "Penta-section Ruler," / "Regular Semi-decagon" which facilitates the quick and precise quinti-section/penta-section of any given angle.

Keywords: Angle penta-section, angle quinti-section, divide angle into five equal parts, compass and straightedge construction, doubling the cube, squaring the circle, angle division.

1. INTRODUCTION

For over three millennia, three classical problems from ancient Greek geometry - "Squaring the Circle", "Doubling the Cube", and "Trisecting An Angle" - have tested the ingenuity of mathematicians. Formulated under the strict constraint of using only an unmarked straightedge and compass, these problems were ultimately declared unsolvable within the standard framework of Euclidean construction. In the nineteenth century, foundational impossibility results by Pierre Wantzel and Ferdinand Von Lindemann, employing algebraic field theory and transcendental number theory, appeared to settle the matter: Wantzel's 1837 theorem ruled out arbitrary angle trisection and cube duplication by straightedge-and-compass methods, and Lindemann's proof of the transcendence of π (1882) made classical squaring of the circle impossible. [4], [5], [2].

Although these algebraic impossibility theorems are rigorous within their respective algebraic and arithmetic frameworks, they depart from the purely constructive spirit of the ancient geometric problems. Classical Greek geometry is fundamentally geometric and constructive in nature - eschewing algebraic or transcendental reasoning - and this difference in perspective motivates a re-evaluation of the ancient challenges strictly from within Euclidean geometric methods. [2], [3], [18] and [19].

This article asserts that the problem of doing penta-section of an arbitrary angle can be resolved within the realm of Euclidean geometry using only a straightedge and compass. The construction presented here follows the classical methods of analysis and synthesis: in the analytic phase the construction is assumed and worked backward to reduce the problem to known solvable configurations, and in the synthetic phase the solution is reconstructed step by step from the initial data.

The resulting construction of the Analysis & Synthesis gives a complete and rigorous solution to the angle penta-section problem; although the original method is intricate, it establishes the feasibility of exact penta-section by classical means.

The previous research further leverages this foundation to produce a substantially simplified, exact construction for trisecting an arbitrary angle - again, strictly within the compass-and-straightedge paradigm. [9], [10]. During this work a novel geometric implement - the Trisection Ruler - was devised. This instrument enables rapid and precise trisection of any given angle and represents a practical advancement in constructive geometry.

In addition, the investigations were guided by an aesthetic and philosophical principle inspired by Lao Tzu's aphorism from the Tao Te Ching: *"The Great Tao is simple, very simple"* (大道至简). Emphasizing simplicity and concentric structure, the author develops exact geometric constructions not only for angle trisection but also for the classical problems of squaring the circle and doubling the cube, relying exclusively on geometric reasoning. [6], [10].

Related publications include the paper "Exact Solution to the Squaring the Circle Problem" (SJPMS, 2024), which presents a construction claimed to produce a square whose area equals that of a given circle solely by Euclidean means and challenges the conventional interpretation of Lindemann's impossibility result. The inverse problem, "Circling the Square," was also published in 2024, and these methods were extended to a new problem, "Circling the Regular Hexagon." The latter problem—apparently not previously treated in the literature—considers the construction of a circle concentric with a given regular hexagon that has the same area. The proposed method identifies a regular dodecagon whose twelve vertices lie on the same circle and on the extended sides of the hexagon; inscribing this dodecagon by straightedge and compass yields a circle whose area equals that of the hexagon. To facilitate this approach a specialized instrument, the Regular Dodecagon Ruler, is introduced. [6], [7], [16], [17].

Beyond technical constructions, this research contributes to the philosophical discussion about mathematical truth and the role of framework in determining what is "possible" or "impossible." While impossibility theorems are decisive within the logical systems and assumptions that underpin them, the results presented here suggest that alternative constructive approaches - consistent with the original spirit of Euclidean geometry - may produce exact solutions previously regarded as unattainable. This perspective aligns with Karl Popper's philosophy of science, in which knowledge is provisional and open to falsification by new evidence. [1].

The remainder of this paper presents the analytic reductions, synthetic constructions, proofs of correctness, and descriptions of the Penta-section Ruler / Regular Semi Decagon. Detailed diagrams and stepwise compass-and-straightedge procedures are provided to demonstrate the exactness and reproducibility of the constructions.

2. PROPOSITIONS

For a given angle $\widehat{\mathbf{UOV}}$, one can use a straightedge and compass:

* to easily construct its bisector.

* to locate $\widehat{\mathbf{UOV}}$ into a vertical position, as Figures below.

By standard definition, the four rays that divide a given angle into five equal sub-angles are called *Penta-sectors*.

By reference [23], there existed a geometrical method to construct a regular decagon inscribed in a circle. Therefore, the follow definition can be created.

2.1 Penta-section Ruler definition:

Given a vertical angle $\widehat{\text{UOV}} < 180°$ then the regular semi-decagon, of which its large base perpendicular to the bisector of $\widehat{\text{UOV}}$, is defined as a *Penta-section Ruler* at an arbitrary line section p = OA on side OU (Figure 1).

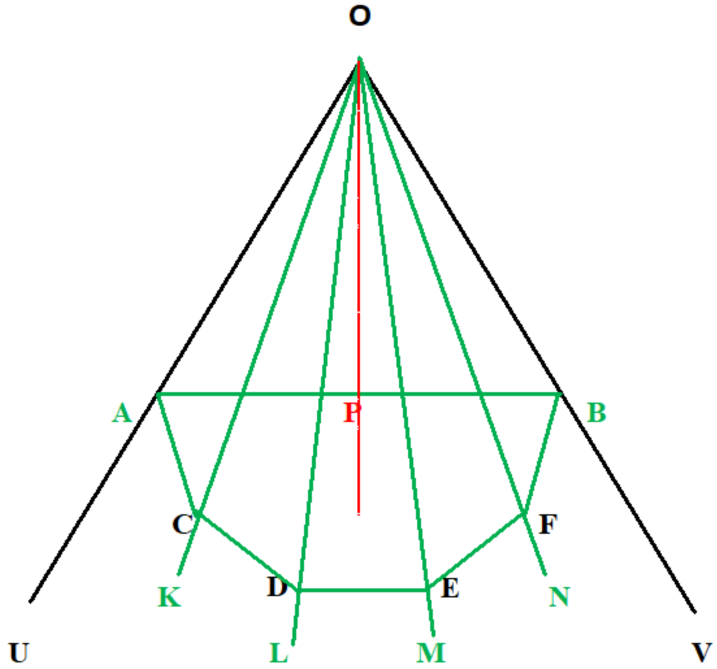


Figure 1: The upside down regular semi-decagon ACDEFB (Penta-section Ruler) and four penta-sectors OK, OL, OM and ON of the given angle $\widehat{UOV}$.

NOTE that point A, OA is an arbitrary length p. Therefore, there exists an infinite number of Penta-section Rulers for a given angle $\widehat{\mathbf{UOV}}$.

ANALYSIS PROPOSITION

2.2 Angle Penta-section Ruler Theorem:

Assume a given angle $\widehat{UOV}$ < 180°, has four penta-sectors OK, OL, OM and ON. Then a Penta-section Ruler for $\widehat{UOV}$ can be constructed using only a straightedge and compass.

NOTE: 'Aim to prove:

- With Geometric Analysis, one can use a straightedge and compass to construct a line segment AB which is perpendicular to the bisector of the given angle and the centre P of AB.
- The Circle (P, r = AP) intersects the 4 penta-sectors of the given angle at points C, D, E, F. Then the upside-down regular semi-decagon ACDEFB matches the Penta-section Ruler definition above, and then is the Penta-section Ruler of the given angle.

PROOF:

Assume $\widehat{\mathbf{UOV}}$ is a given angle with its four penta-sectors, OK, OL, OM, and ON. Let A be an arbitrary point on the left side of the given angle. Draw line AB, which is perpendicular to the angle bisector at P (**Figure 2a** below). Draw the circle (O, OA = p) that intersects the above four penta-sectors at G, H, I, and J. Then draw the semi-circle (P, PA) that intersects the above four penta-sectors OK, OM, ON, and OL at C, D, E, and F (**Figure 2a** below).

By the analysis assumption of this theorem, we consider point C on the bisector OK (red colour) of the angle $\widehat{\mathbf{UOD}}$. From C, draw a circle (C, CA) that intersects the circle (O, OA) at A & H, and OH is symmetrical to OA/OU and OH must cut (C, CA) at D' where D'H is symmetrical to AQ, because OK is perpendicular chord AH of (C, CA). This symmetry property shows D' is overlapped D and implies

CA = CD (1)

Then, from D draw circle (D, DC) for **Figure 2b** below and make a similar proof to show

CD = DE (2)

Then, the similar proofs with circles centred E and F, show:

DE = EF (3)

EF = FB (4)

By the Penta-section Ruler definition above and (1), (2), (3) & (4), ACDEFB is a penta-section ruler of the given $\widehat{UOV}$, as required.

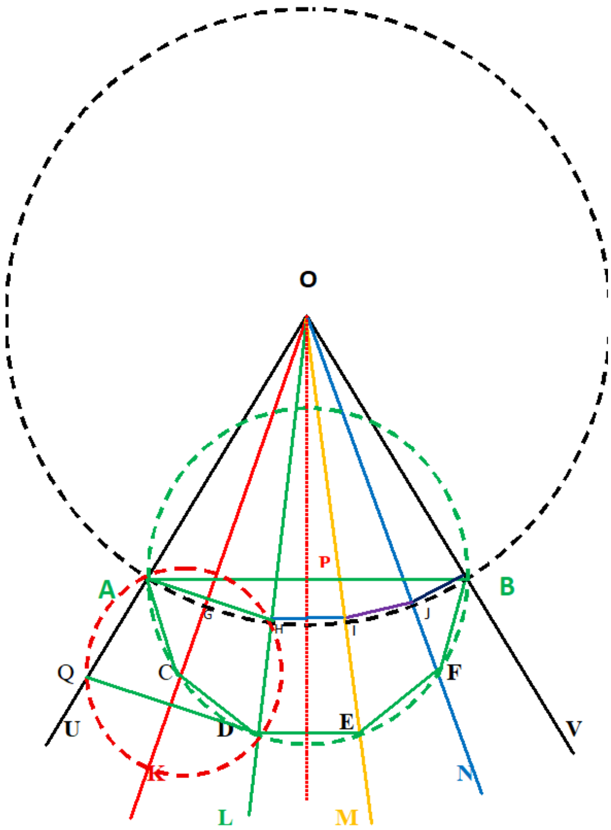


Figure 2a: The given angle $\widehat{UOV}$, its penta-sectors OK, OL, OM & ON, dashed circles (O, OA), (P, PA) and (C, CA)

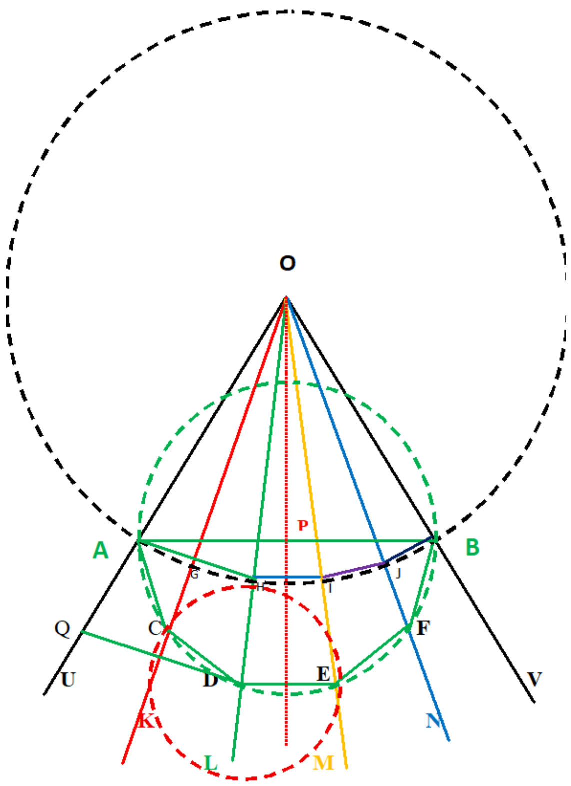


Figure 2b: ***Penta-section Ruler ABCDEF, and the circle (D, DC).***

SYNTHESIS PROPOSITION

2.3 Angle Penta-section Theorem:

Let $\widehat{UOV} < 180°$, be a given angle with its penta-section ruler ACDEFB, where OA = an arbitrary length p. Then from this penta-section ruler, the angle's four PENTA-SECTORS OK, OL, OM & ON can be constructed exactly and accurately using only a straightedge and compass.

NOTE: 'Aim to prove*:*

- With Geometric Synthesis, one can use a **straightedge and compass** to construct
  - a line segment AB, which is perpendicular to the bisector of a given angle and the centre P of AB.
  - The Circle (P, r = AP) has an upside-down regular semi-decagon ACDEFB which intersects the four penta-sectors of the angle at points C, D, E & F.
- Rays OK, OL, OM, and ON are the four penta-sectors of the given angle.

PROOF:

Let $\widehat{\mathbf{UOV}}$ be a given angle to be penta-sectioned, and let A be an arbitrary point on side OU such that OA = p, where p is an arbitrary length.

- Using only a straightedge and compass, construct line AB perpendicular to the angle bisector at point P (see **Figure 3a**). Also construct a penta-section ruler **ACDEFB** using straightedge and compass [20 & 21]. Draw line **OC**, which intersects the circle centered at **O** with radius **OA = p** at point **G** (**Figure 3a**). Next, draw the circle centered at **C** with radius **CA**, which intersects the circle centered at **O** (radius **OA**) at point **H**. Then, from the bisector OG of the angle $\widehat{\mathbf{AOH}}$, we get

GA = GH (1).

Since **OA**, **OG**, and **OH** are radius of the circle centered at **O**, and from (1), triangles **AOG** and **GOH** are equal & congruent (by Elementary Geometry). Therefore, the corresponding central angles are equal, and

$\widehat{\mathbf{AOG}}$ / $\widehat{UOK}$ = $\widehat{\mathbf{GOH}}$ / $\widehat{KOL}$ (2).

As the two circles (O, OA) & (C, CA) intersect at A & H, and the circle (C, CA) cuts OU at Q, then the symmetric al line segment of AQ via the axe OK (bisector of $\widehat{AOH}$), which is HD', must be in line with OH and must be an opposite side of the inscribed & isosceles trapezoid AQD'H. Property of the isosceles trapezoid and the equality (2) above imply that D'H is overlapped HD or line **OH** intersects the circle centered at **P** with radius **PA** at point **D**.

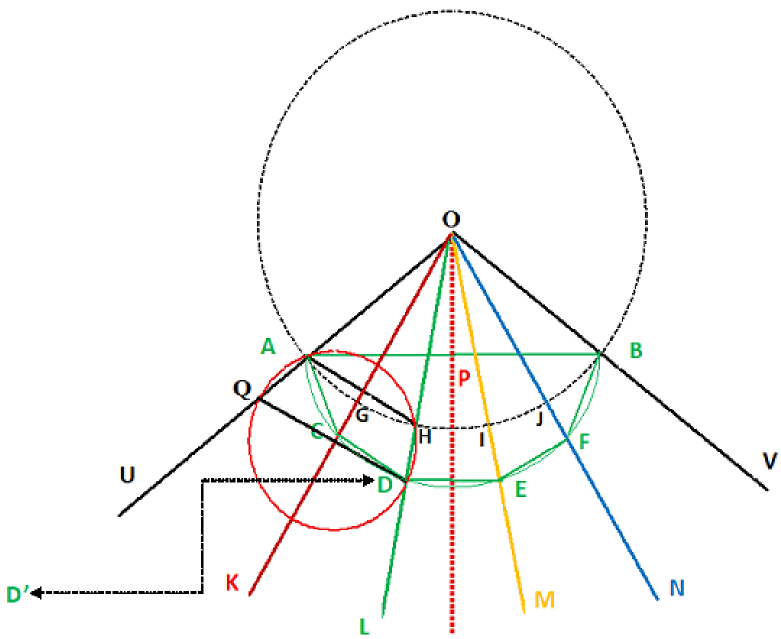


Figure 3a: Penta-section Ruler ABCDEF, the given angle $\widehat{\mathbf{UOV}}$, and the first angle $\widehat{\mathbf{AOC}}$ with it's right hand side OG intersecting vertex C of the Ruler

- Now consider the circle centered at D, radius DG such that this circle intersects (O, OA) at I. The intersection of (D, DG) and (O, OA) results two triangles OGD and OID are equal and exactly congruent (**Figure 3b**). By the symmetry, equality, and congruence of the two triangles OGD & OID, right side OI of triangle OID cuts the penta-ruler ABCDEF at point E, as required. Therefore,

$\widehat{\mathbf{AOC}}$ = $\widehat{\mathbf{COD}}$ = $\widehat{\mathbf{DOE}}$ (3)

Then, draw the bisector of the angle $\widehat{\mathbf{EOB}}$, (**Figure 3c**). By the symmetry, this bisector intersects the penta-ruler ABCDEB at point F, which is the symmetrical point D via the bisector (red dashed line) of the given angle $\widehat{UOV}$. This result and (3) create

$$\widehat{AOC} = \widehat{COD} = \widehat{EOF} = \widehat{FOB} \qquad (4)$$

From results (2), (3), and (4), it follows that the five angles $\widehat{UOK}$, $\widehat{KOL}$, $\widehat{LOM}$, $\widehat{MON}$ and $\widehat{NOV}$ are equal.

Thus, the original angle $\widehat{UOV}$ is divided into five equal parts. The rays drawn from $\mathbf{O}$ through the successively constructed points $\mathbf{K}, \mathbf{L}, \mathbf{M}, \mathbf{N}$ form four interior rays that partition $\widehat{UOV}$ into five equal sub-angles.

Therefore, any angle less than 180°can be divided into five equal parts using only a straightedge and compass.

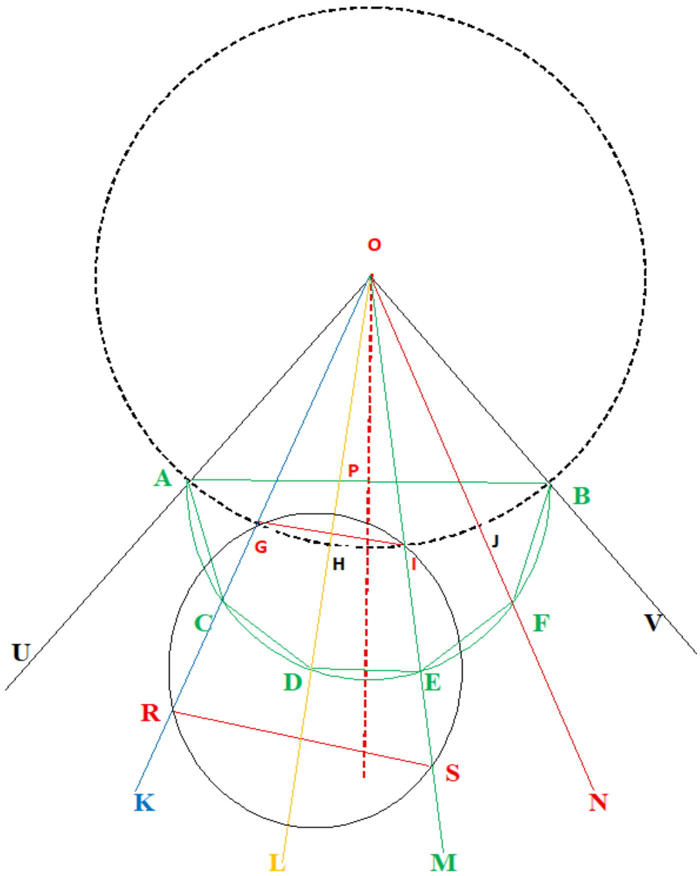


Figure 3b: Penta-section Ruler ABCDEF, the given angle $\widehat{UOV}$, and the first angle $\widehat{AOC}$ with it's right hand side OG intersecting vertex C of the Ruler

Summary of Research Findings / Method

This research establishes several significant findings in the classical problem of angle penta-section using only a compass and straightedge within the framework of Euclidean Geometry:

1. **Exact Construction of Angle Penta-sectors**:

It is possible to construct four defined penta-sectors / quinti-sectors of any given angle exactly, using only a compass and a straightedge, within the principles of classical Euclidean geometry.

2. **Innovation of the Penta-section Ruler**:

The core advancement enabling this construction is the introduction of a conceptual tool termed the Penta-section Ruler / Quinti-section Ruler. This innovation allows the application of classical geometric techniques to address the historically unsolved problem of angle trisection.

3. **Geometric Nature of the Penta-section Ruler**:

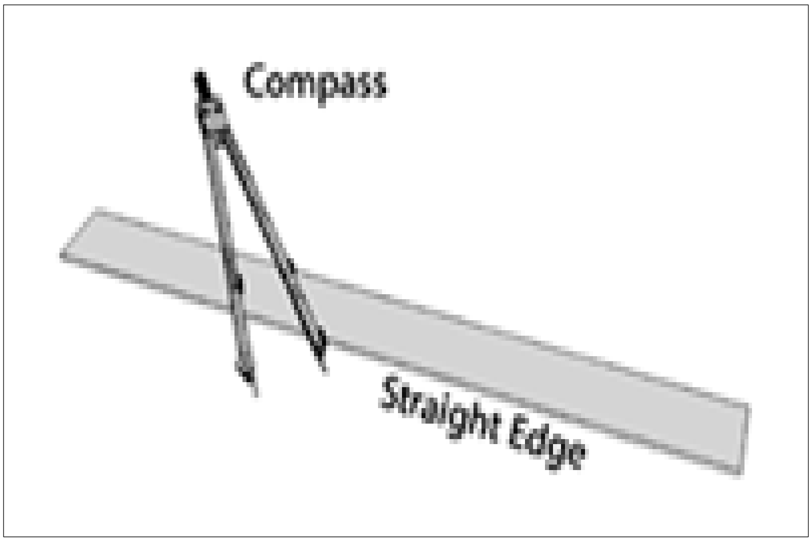
The Penta-section Ruler may be interpreted as a geometric parameter, dependent on the size of the given angle and an arbitrary parameter p, defining two lengths on either side of the angle's vertex.

4. **Refutation of Wantzel's Impossibility Proof**:

This method serves as a counterexample to Pierre Wantzel's 1837 algebraic proof of the impossibility of angle trisection using classical tools. By providing a constructive solution, this result challenges the scope and assumptions of Wantzel's argument.

5. Main Theorem (Angle Penta-section Theorem)

Any given angle can be divided into five equal sub-angles, exactly and accurately, using only a compass and a straightedge, within the axioms of Euclidean geometry.



Completed on 23 March 2026 in the UK.

3. DISCUSSION & CONCLUSION

3.1 DISCUSSION

Innovation Cannot Thrive When Constrained by Traditional Theories

The above problem of 'dividing a given angle into five equal parts' using only a straightedge and compass—referred to as the "*Penta-section of an Angle*"—did not previously arise in classical geometric construction. It emerged only after I resolved the long-standing challenge of angle trisection and published the solution last year.

In the past, those who entered the construction industry typically gained some understanding of its history. One notable figure is Joseph Monier (1823–1906), the inventor of reinforced concrete. He first presented his invention at the Paris Exhibition in 1867 and was granted the world's first patent for reinforced concrete. Subsequently, he received additional patents for reinforced concrete pipes, tanks, beams, and other applications. The first reinforced concrete bridge was also built according to his design. However, Monier's invention was not initially recognized by construction engineers and leading experts in France and around the world. Bound by conventional theories that treated steel and concrete as separate materials, and lacking knowledge of their combined potential, they doubted the durability of the composite material. Furthermore, due to Monier's status as a common individual rather than an academic or professional insider, his work was largely disregarded. As a result, meaningful application of his invention was delayed until the late 19th and early 20th centuries *(nearly 30 years delay!)*. Nevertheless, reinforced concrete eventually became one of the greatest innovations in human history, revolutionizing the construction industry. This breakthrough is attributed to Monier, a self-taught inventor whose ideas ultimately prevailed despite initial resistance. His success was made possible by a few individuals in the engineering community who recognized the invention's value and either acquired the rights or continued to develop it. Today, it is widely acknowledged that without reinforced concrete, it would be impossible to construct skyscrapers, strong bridges, modern highways with overpasses and underpasses, and the vast infrastructure required for large contemporary cities. A similar example can be seen in the invention of Blockchain technology by Satoshi Nakamoto—an individual whose identity remains unknown due to a deliberate choice to remain anonymous. Like Monier, Nakamoto's work has had a profound impact on the world, despite coming from outside traditional academic or institutional frameworks. These examples demonstrate that strict adherence to established theories can hinder creativity and innovation. True progress often originates from those willing to think beyond conventional boundaries and from those who dare to explore uncharted territory. Another example of great invention is the Blockchain technology of Satoshi Nakamoto, a person who has a name but no one knows who he is.

In the past, knowledge was often considered scientific if it could be confirmed through specific evidence or experiments. However, Karl Popper, in his book Logik der Forschung (The Logic of Scientific Discovery), published in 1934, demonstrated that a fundamental characteristic of scientific hypotheses is their ability to be proven wrong (falsifiability). Anything that cannot be refuted by evidence is temporarily regarded as true until new evidence emerges. For instance, in astronomy, the Big Bang theory is widely accepted, but in the future, anyone who discovers a flaw in this theory will be acknowledged by the entire physics community. Furthermore, no scientific theory lasts forever; rather, it is specific research and discoveries that continually build upon one another [1].

Moreover, PEOPLE MAY STAND STILL – BUT THE EARTH DOES NOT. In 1851, in the nave of the Panthéon, Léon Foucault conducted a quiet yet revolutionary experiment. He suspended from the dome a pendulum 67 meters in length. As it oscillated, observers noticed that the plane of its swing gradually rotated over time. The pendulum itself did not change direction; rather, the Earth was rotating beneath it.

With this simple but profound demonstration, Foucault provided direct, visible evidence that the Earth spins on its axis. Prior to this experiment, the heliocentric model proposed by Nicolaus Copernicus and later supported by Galileo Galilei had already established the theory of Earth's rotation. However, these conclusions rested primarily on mathematical reasoning and astronomical observation. Foucault sought something more immediate: empirical proof accessible to all.

No complex equations were required. No debate was necessary. One needed only to stand and observe. For the first time in history, people could witness with their own eyes that they inhabited a moving planet.

Foucault did more than demonstrate a physical phenomenon; he transformed humanity's perception of its place in the universe. He made the invisible visible. In an age of intellectual contention, he chose demonstration over argument, evidence over rhetoric.

This principle extends beyond physics. In contemporary society, opinions are abundant, and debates are constant. Many claim to possess ideas, ambitions, and potential. Yet ideas alone do not alter reality. The world does not revolve around assertions; it advances through action.

Foucault's pendulum did not persuade because he spoke about it—it persuaded because it moved.

If one believes oneself capable of meaningful achievement, the path forward is not endless discussion but deliberate creation. As the ancient philosopher Laozi (Lao Tzu) expressed, "The Great Tao is simple." Simplicity, however, does not imply passivity. It calls for clarity of purpose and decisive effort: build something tangible, write with substance, develop expertise, initiate a project—and substantiate claims with results.

When work is visible and measurable, validation becomes unnecessary. People will not ask whether you are capable; they will observe what you have created and draw their own conclusions.

We live on a planet in constant motion. The Earth continues to rotate, indifferent to hesitation. So too do opportunities evolve and pass. The essential question, then, is not whether the world moves—but whether we move with it.

The lesson drawn from great figures is not merely admiration of their achievements. It is the recognition that decisive action transforms theory into experience, and potential into reality. It looks like the Newton's apple showed people the visible gravity force. The opportunity to begin remains—always—today, as this article transformed *an invisible penta-sectioned angle into the visible penta-sectioned angle.*

Starting from accepted premises, without proof, one uses deductive reasoning to arrive at theorems and corollaries. With different premises, we develop different mathematical systems. For example, the premise "from a point outside a line, only one parallel line can be drawn to the given line" leads to Euclidean geometry. If we assume that no parallel lines can be drawn from that point, we enter the realm of Riemannian geometry. Alternatively, Lobachevskian geometry assumes that an infinite number of parallel lines can be drawn through that point. No scientific theory lasts forever, but specific research and discoveries continuously build upon each other. The three classic ancient Greek mathematical challenges likely referring to are "Doubling The Circle", "Trisecting An Angle" and "Squaring The Circle", all famously proven Impossible under strict compass-and-straightedge constraints, by Pierre Wantzel (1837) using field theory and algebraic methods, then also by Ferdinand von Lindemann (1882) after proving π is transcendental. These original Greek challenges remain impossible under classical rules since their proofs rely on deep algebraic/transcendental properties settled in the 19th century. Recent claims may involve reinterpretations or unrelated advances but do overturn these conclusions above. Among these, the "Squaring A Circle" problem and related problems involving π have captivated both professional and amateur mathematicians for millennia.

3.2 Conclusion

Most mathematicians and mathematics enthusiasts accept that the three classical Greek problems—*Squaring the Circle***,** *Doubling the Cube***,** *and Trisecting an Angle* - are impossible to solve using only a straightedge and compass. This consensus largely stems from the work of Pierre Wantzel (1837), who employed field theory and algebraic methods to prove the impossibility of certain geometric constructions. However, it is important to recognize that "***Squaring the Circle***" is fundamentally a geometric construction problem, and the algebraic approach may not fully address the nuances of Euclidean geometry. Further support for the impossibility of squaring the circle came from Ferdinand von Lindemann's proof in 1882 that π is transcendental. From this, it is commonly concluded that since π cannot be constructed using a finite sequence of straightedge and compass steps, it is impossible to square the circle in the classical sense.

However, it is worth considering a different perspective. If one can construct a square of area A equal to that of a given circle (with centre O and radius r), then, using the formula $A = \pi r^2$, one could derive a numerical precision of π by measuring the square's edge precisely with modern laser technology. From this measurement, π could be computed using the fact that π is equal accurately to A divided by r^2.

I hold a different point: *I believe I have constructed a valid solution to the "**Squaring the Circle"** problem using only a straightedge and compass, in accordance with the classical constraints and published* (see References)**.** This belief strengthens my resolve as I pursue a solution to the *Angle Penta-section/Angle Quinti-section* problem. The techniques of *geometrical analysis and synthesis* are instrumental in this effort.

Suppose we are given an angle $U\hat{O}V$, that we wish to divide exactly into five equal small angles $U\hat{O}K$, $K\hat{O}L$, $L\hat{O}M$, $M\hat{O}N$, and $N\hat{O}V$. By analyzing the relationships among the angle's parts and constructing a circle with diameter AB, where the line AB perpendicular to the bisector of the angle $\mathbf{U\hat{O}V}$ and intersects the angle sides at points A & B, we can demonstrate that segment AB functions as a *Penta-**section Ruler's key***. This aids in achieving an accurate and geometrically valid penta-section (Geometry Analysis). Continuing with the synthetic approach: *from angle* $\mathbf{U\hat{O}V}$*, select any point A on one side and draw line AB perpendicular to the bisector of* $\mathbf{U\hat{O}V}$. Then, construct a circle centred at point P with radius ½AB. Through this logical progression, we develop the necessary ***Penta-section Ruler*** and establish a foundation for a rigorous proof, which will be presented in detail in the Proof of Angle Penta-section Theorem, Definition of the Penta-section Ruler and Angle Penta-section Method in **Section 2** above (Geometry Synthesis). Additionally, this study opens the door to new investigations into the "*Devide Into Seven Equal Parts For A Given Angle" / "**Septi-section Of An Angle"*** problem within Euclidean geometry, once again using only a straightedge and compass. Moreover, further investigation solutions can be researched for the Angle Multi-section in the near future.

It is difficult to realise why the "Angle Penta-section" challenge above has not existed before this published article, meanwhile its solution within the classic Euclidean Geometry is proved simply as above.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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