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Research Article

Classification of All Single Traveling Wave Solutions to the Gardner-KP Equation

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Abstract: Under the travelling wave transformation, some nonlinear partial differential equations such as the Gardner-KP equation are transformed to ordinary differential equation. Then, the classifications of all single traveling wave solution to this equation are obtained.
Keywords: the nonlinear partial differential equation; complete discrimination system for polynomial; traveling wave transform; the Gardner-KP equation

INTRODUCTION

Nonlinear partial differential equations(PDE) in mathematical physics play a major role in various fields. Searching for the exact solutions of the nonlinear PDE has become an important problem in nonlinear science. Not only can these exact solutions describe many important phenomena in physics and other fields, but they can also help physicists to understand the mechanisms of the complicated physical phenomena.

A variety of powerful methods have been employed to study the nonlinear PDE, such as the inverse scattering transform[1], the Bäcklund transformation method[2], the Darboux transformation[3], Hirota's bilinear method[4], the homogeneous balance method[5], the tanh function method[6], sine-cosine method[7], the exp-function method[8], the G'/G- expansion method[9], and so on. Recently, Professor Liu used the theory of complete discrimination system for polynomial[10-12] to find exact solutions to many nonlinear differential equations.

In this article, we used the theory of complete discrimination system for polynomial to investigate the positive and negative models of the Gardner-KP equations[13-14] given by

$$(u_t + 6uu_x \pm 6u^2 u_x + u_{xxx})_x + u_{yy} = 0. \quad (1)$$

REDUCTION

In this part,we use positive Gardner-KP equation as an example to explain the application of the method for the sake of simplicity.

$$(u_t + 6uu_x + 6u^2 u_x + u_{xxx})_x + u_{yy} = 0. \quad (2)$$

Taking the traveling wave transformation $u = u(\xi_1)$ and $\xi_1 = kx + ly + \omega t$, we can obtain the corresponding ODE

$$k(\omega u' + 6kuu' + 6ku^2 u' + k^3 u''')' + l^2 u'' = 0. \quad (3)$$

Integrating Eq.(3) with respect to ξ_1 twice, we simplify it and yield

$$u'' = a_3 u^3 + a_2 u^2 + a_1 u + a_0. \quad (4)$$

Where $a_3 = -\frac{2}{k^2}, a_2 = -\frac{3}{k^2}, a_1 = \frac{c_1 - k\omega - l^2}{k^4}, a_0 = \frac{c_0}{k^4}, c_0$ and c_1 are integral constants.

From Eq.(4),we get

$$(u')^2 = \frac{1}{2} a_3 u^4 + \frac{2}{3} a_2 u^3 + a_1 u^2 + 2a_0 u + d. \quad (5)$$

We notice $a_3 < 0$ and take transformations as follows

$$w=(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3}),\xi=(-\frac{1}{2}a_3)^{\frac{1}{4}}\xi_1. \quad (6)$$

Then Eq.(5) becomes

$$(w')^2=-(w^4+pw^2+qw+r). \quad (7)$$

Where w is a function of ξ and

$$p=-a_1(-\frac{1}{2}a_3)^{-\frac{1}{2}}+\frac{1}{3}(-\frac{1}{2}a_3)^{-\frac{1}{2}}a_2^2a_3^{-1}.$$

$$q=\frac{2}{3}a_1(-\frac{1}{2}a_3)^{-\frac{1}{4}}a_2a_3^{-1}-\frac{4}{27}(-\frac{1}{2}a_3)^{-\frac{1}{4}}a_2^3a_3^{-2}-2a_0(-\frac{1}{2}a_3)^{-\frac{1}{4}}. \quad (8)$$

$$r=-a_0+\frac{2}{3}a_0a_2a_3^{-1}-\frac{1}{9}a_1a_2^2a_3^{-2}+\frac{1}{54}a_2^4a_3^{-3}.$$

We write the complete discrimination system for polynomial $F(w)=w^4+pw^2+qw+r$ as follows

$$D_1=4, D_2=-p, D_3=8rp-2p^3-9q^2,$$

$$D_4=4p^4r-p^3q^2+36prq^2-32r^2p^2-\frac{27}{4}q^4+64r^3, \quad (9)$$

$$E_2=9q^2-32pr.$$

Then we consider the following ODE

$$(w')^2=\varepsilon(w^4+pw^2+qw+r). \quad (10)$$

Where $\varepsilon=\pm1$. Rewrite Eq.(10) by integral form as follows

$$\pm(\xi-\xi_0)=\int\frac{dw}{\sqrt{\varepsilon(w^4+pw^2+qw+r)}}. \quad (11)$$

CLASSIFICATION

According to the complete discrimination system for the fouth order polynomial, we give the corresponding single traveling wave solutions to Eq.(2).

Case 1. $D_4=0$, $D_3=0$, $D_2<0$. Then we have

$$F(w)=[(w-l_1)^2+s_1^2]^2. \quad (12)$$

Where l_1 and s_1 are real numbers, $s_1>0$.

When $\varepsilon=1$, we have

$$w=s_1\tan[s_1(\xi-\xi_0)]+l_1. \quad (13)$$

The corresponding solution is

$$u=(\frac{1}{2}a_3)^{-\frac{1}{4}}\{s_1\tan[s_1(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)]+l_1\}-\frac{a_2}{3a_3}. \quad (14)$$

Case 2. $D_4=0$, $D_3=0$, $D_2=0$. Then we have

$$F(w)=w^4. \quad (15)$$

When $\varepsilon=1$, we have

$$w=-\frac{1}{(\xi-\xi_0)}. \quad (16)$$

The corresponding solution is

$$u=-(\frac{1}{2}a_3)^{-\frac{1}{2}}\frac{1}{(\xi_1-\xi_0)}-\frac{a_2}{3a_3}. \quad (17)$$

Case 3. $D_4=0$, $D_3=0$, $D_2>0$, $E_2=0$. Then we have

$$F(w)=(w-\alpha)^2(w-\beta)^2. \quad (18)$$

Where α and β are real numbers, $\alpha > \beta$.

When $\varepsilon = 1$

(i) If $w > \alpha$ or $w < \beta$, we have

$$w = \frac{\beta-\alpha}{2}[\coth\frac{\alpha-\beta}{2}(\xi-\xi_0)-1]+\beta. \tag{19}$$

The corresponding solution is

$$u = (\frac{1}{2}a_3)^{-\frac{1}{4}}\left\{\frac{\beta-\alpha}{2}[\coth\frac{\alpha-\beta}{2}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)-1]+\beta\right\}-\frac{a_2}{3a_3}. \tag{20}$$

(ii) If $\alpha > w > \beta$, we have

$$w = \frac{\beta-\alpha}{2}[\tanh\frac{\alpha-\beta}{2}(\xi-\xi_0)-1]+\beta. \tag{21}$$

The corresponding solution is

$$u = (\frac{1}{2}a_3)^{-\frac{1}{4}}\left\{\frac{\beta-\alpha}{2}[\tanh\frac{\alpha-\beta}{2}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)-1]+\beta\right\}-\frac{a_2}{3a_3}. \tag{22}$$

Case 4. $D_4 = 0$, $D_3 > 0$, $D_2 > 0$. Then we have

$$F(w) = (w-\alpha)^2(w-\beta)(w-\gamma). \tag{23}$$

Where α, β, γ are real numbers, and $\beta > \gamma$.

When $\varepsilon = 1$

(i) If $\alpha > \beta$, $w > \beta$ or if $\alpha < \gamma$, $w < \gamma$, we have

$$\pm(\xi-\xi_0) = \frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{[\sqrt{(w-\beta)(\alpha-\gamma)}-\sqrt{(\alpha-\beta)(w-\gamma)}]^2}{|w-\alpha|}. \tag{24}$$

The corresponding solution is

$$\pm(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0) = \frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{\left\{\sqrt{[(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\beta](\alpha-\gamma)}-\sqrt{(\alpha-\beta)[(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\gamma]}\right\}^2}{\left|(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\alpha\right|}. \tag{25}$$

(ii) If $\alpha > \beta$, $w < \gamma$ or if $\alpha < \gamma$, $w < \beta$, we have

$$\pm(\xi-\xi_0) = \frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{[\sqrt{(w-\beta)(\gamma-\alpha)}-\sqrt{(\beta-\alpha)(w-\gamma)}]^2}{|w-\alpha|}. \tag{26}$$

The corresponding solution is

$$\pm(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0) = \frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{\left\{\sqrt{[(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\beta](\gamma-\alpha)}-\sqrt{(\beta-\alpha)[(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\gamma]}\right\}^2}{\left|(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\alpha\right|}. \tag{27}$$

(iii) If $\beta > \alpha > \gamma$, we have

$$\pm(\xi-\xi_0) = \frac{1}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}}\arcsin\frac{(w-\beta)(\alpha-\gamma)+(\alpha-\beta)(w-\gamma)}{|(w-\alpha)(\beta-\gamma)|}. \tag{28}$$

The corresponding solution is

$$\pm(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)=\frac{1}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}}\arcsin\frac{[(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\beta](\alpha-\gamma)+(\alpha-\beta)[(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\gamma]}{\left|[(\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\alpha](\beta-\gamma)\right|}. \tag{29}$$

When $\varepsilon=-1$

(i) If $\alpha>\beta$, $w>\beta$ or if $\alpha<\gamma$, $w<\gamma$, we have The corresponding solution is

$$\pm(\xi-\xi_0)=\frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{[\sqrt{(w-\beta)(\alpha-\gamma)}-\sqrt{(\alpha-\beta)(w-\gamma)}]^2}{|w-\alpha|}. \tag{30}$$

The corresponding solution is

$$\pm(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)=\frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{\left\{\sqrt{[(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\beta](\alpha-\gamma)}-\sqrt{(\alpha-\beta)[(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\gamma]}\right\}^2}{\left|(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\alpha\right|}. \tag{31}$$

(ii) If $\alpha>\beta$, $w<\gamma$ or if $\alpha<\gamma$, $w<\beta$, we have

$$\pm(\xi-\xi_0)=\frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{[\sqrt{(w-\beta)(\gamma-\alpha)}-\sqrt{(\beta-\alpha)(w-\gamma)}]^2}{|w-\alpha|}. \tag{32}$$

The corresponding solution is

$$\pm(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)=\frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}}\ln\frac{\left\{\sqrt{[(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\beta](\gamma-\alpha)}-\sqrt{(\beta-\alpha)[(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\gamma]}\right\}^2}{\left|(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\alpha\right|}. \tag{33}$$

(iii) If $\beta>\alpha>\gamma$, we have

$$\pm(\xi-\xi_0)=\frac{1}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}}\arcsin\frac{(w-\beta)(\alpha-\gamma)+(\alpha-\beta)(w-\gamma)}{|(w-\alpha)(\beta-\gamma)|}. \tag{34}$$

The corresponding solution is

$$\pm(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)=\frac{1}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}}\arcsin\frac{[(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\beta](\alpha-\gamma)+(\alpha-\beta)[(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\gamma]}{\left|[(-\frac{1}{2}a_3)^{\frac{1}{4}}(u+\frac{a_2}{3a_3})-\alpha](\beta-mma)\right|}. \tag{35}$$

Case 5. $D_4=0$, $D_3=0$, $D_2>0$, $E_2=0$. Then we have

$$F(w)=(w-\alpha)^3(w-\beta). \tag{36}$$

Where α and β are real numbers.

When $\varepsilon=1$, if $w>\alpha$, $w>\beta$ or if $w<\alpha$, $w<\beta$, we have

$$w=\frac{4(\alpha-\beta)}{(\alpha-\beta)^2(\xi-\xi_0)^2-4}. \tag{37}$$

The corresponding solution is

$$u=(\frac{1}{2}a_3)^{-\frac{1}{4}}\frac{4(\alpha-\beta)}{(\alpha-\beta)^2(\frac{1}{2}a_3)^{\frac{1}{2}}(\xi_1-\xi_0)^2-4}-\frac{a_2}{3a_3}. \tag{38}$$

When $\varepsilon=-1$,if $w>\alpha$, $w<\beta$ or if $w<\alpha$, $w>\beta$, we have

$$w = \frac{4(\beta-\alpha)}{(\alpha-\beta)^2(\xi-\xi_0)^2+4}. \tag{39}$$

The corresponding solution is

$$u = (-\frac{1}{2}a_3)^{-\frac{1}{4}} \frac{4(\beta-\alpha)}{(\alpha-\beta)^2(-\frac{1}{2}a_3)^{\frac{1}{2}}(\xi_1-\xi_0)^2+4} - \frac{a_2}{3a_3}. \tag{40}$$

Case 6. $D_4 = 0$, $D_2 D_3 < 0$. Then we have

$$F(w) = (w-\alpha)^2[(w-l_1)^2 + s_1^2]. \tag{41}$$

where α, l_1 and s_1 are real numbers.

When $\varepsilon = 1$, we have

$$w = \frac{\exp[\pm\sqrt{(\alpha-l_1)^2+s_1^2}(\xi-\xi_0)] - \gamma + \sqrt{(\alpha-l_1)^2+s_1^2}}{\{\exp[\pm\sqrt{(\alpha-l_1)^2+s_1^2}(\xi-\xi_0)] - \gamma\}^2 - 1}. \tag{42}$$

The corresponding solution is

$$u = (\frac{1}{2}a_3)^{-\frac{1}{4}} \frac{\exp[\pm\sqrt{(\alpha-l_1)^2+s_1^2}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)] - \gamma + \sqrt{(\alpha-l_1)^2+s_1^2}}{\{\exp[\pm\sqrt{(\alpha-l_1)^2+s_1^2}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0)] - \gamma\}^2 - 1} - \frac{a_2}{3a_3}. \tag{43}$$

where $\gamma = \frac{\alpha-2l_1}{\sqrt{(\alpha-l_1)^2+s_1^2}}$.

Case 7. $D_4 > 0$, $D_3 > 0$, $D_1 > 0$. Then we have

$$F(w) = (w-\alpha_1)(w-\alpha_2)(w-\alpha_3)(w-\alpha_4). \tag{44}$$

where α_1, α_2, α_3 and α_4 are real numbers, and $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$.

When $\varepsilon = 1$

(i) If $w > \alpha_1$ or $w < \alpha_4$, we have

$$w = \frac{\alpha_2(\alpha_1-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi-\xi_0), m\right) - \alpha_1(\alpha_2-\alpha_4)}{(\alpha_1-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi-\xi_0), m\right) - (\alpha_2-\alpha_4)}. \tag{45}$$

The corresponding solution is

$$u = (\frac{1}{2}a_3)^{-\frac{1}{4}} \frac{\alpha_2(\alpha_1-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0), m\right) - \alpha_1(\alpha_2-\alpha_4)}{(\alpha_1-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0), m\right) - (\alpha_2-\alpha_4)} - \frac{a_2}{3a_3}. \tag{46}$$

(ii) If $\alpha_2 > w > \alpha_3$, we have

$$w = \frac{\alpha_4(\alpha_2-\alpha_3)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi-\xi_0), m\right) - \alpha_3(\alpha_2-\alpha_4)}{(\alpha_2-\alpha_3)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi-\xi_0), m\right) - (\alpha_2-\alpha_4)}. \tag{47}$$

The corresponding solution is

$$u = (\frac{1}{2}a_3)^{-\frac{1}{4}} \frac{\alpha_4(\alpha_2-\alpha_3)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0), m\right) - \alpha_3(\alpha_2-\alpha_4)}{(\alpha_2-\alpha_3)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0), m\right) - (\alpha_2-\alpha_4)} - \frac{a_2}{3a_3}. \tag{48}$$

When $\varepsilon = -1$

(i) $\alpha_1 > w > \alpha_2$, we have

$$w=\frac{\alpha_3(\alpha_1-\alpha_2)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi-\xi_0),m\right)-\alpha_2(\alpha_1-\alpha_3)}{(\alpha_1-\alpha_2)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi-\xi_0),m\right)-(\alpha_1-\alpha_3)}. \quad (49)$$

The corresponding solution is

$$u=(-\frac{1}{2}a_3)^{-\frac{1}{4}}\frac{\alpha_3(\alpha_1-\alpha_2)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0),m\right)-\alpha_2(\alpha_1-\alpha_3)}{(\alpha_1-\alpha_2)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0),m\right)-(\alpha_1-\alpha_3)}-\frac{a_2}{3a_3}. \quad (50)$$

(ii) $\alpha_3>w>\alpha_4$, we have

$$w=\frac{\alpha_1(\alpha_3-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi-\xi_0),m\right)-\alpha_4(\alpha_1-\alpha_3)}{(\alpha_3-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\xi-\xi_0),m\right)-(\alpha_3-\alpha_1)}. \quad (51)$$

The corresponding solution is

$$u=(-\frac{1}{2}a_3)^{-\frac{1}{4}}\frac{\alpha_1(\alpha_3-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0),m\right)-\alpha_4(\alpha_3-\alpha_1)}{(\alpha_3-\alpha_4)\mathrm{sn}^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0),m\right)-(\alpha_3-\alpha_1)}-\frac{a_2}{3a_3}. \quad (52)$$

where $m^2=\frac{(\alpha_1-\alpha_2)(\alpha_3-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}$.

Case 8. $D_4<0$, $D_2D_3\geq 0$. Then we have

$$F(w)=(w-\alpha)(w-\beta)[(w-l_1)^2+s_1^2]. \quad (53)$$

where α, β, l_1, s_1 are real numbers, and $\alpha>\beta$, $s_1>0$.

When $\varepsilon=1$, we have

$$w=\frac{a\mathrm{cn}\left(\frac{\sqrt{-2s_1m_1(\alpha-\beta)}}{2mm_1}(\xi-\xi_0),m\right)+b}{c\mathrm{cn}\left(\frac{\sqrt{-2s_1m_1(\alpha-\beta)}}{2mm_1}(\xi-\xi_0),m\right)+d}. \quad (54)$$

The corresponding solution is

$$u=(\frac{1}{2}a_3)^{-\frac{1}{4}}\frac{a\mathrm{cn}\left(\frac{\sqrt{-2s_1m_1(\alpha-\beta)}}{2mm_1}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0),m\right)+b}{c\mathrm{cn}\left(\frac{\sqrt{-2s_1m_1(\alpha-\beta)}}{2mm_1}(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0),m\right)+d}-\frac{a_2}{3a_3}. \quad (55)$$

When $\varepsilon=-1$, we have

$$w=\frac{a\mathrm{cn}\left(\frac{\sqrt{2s_1m_1(\alpha-\beta)}}{2mm_1}(\xi-\xi_0),m\right)+b}{c\mathrm{cn}\left(\frac{\sqrt{2s_1m_1(\alpha-\beta)}}{2mm_1}(\xi-\xi_0),m\right)+d}. \quad (56)$$

The corresponding solution is

$$u=(-\frac{1}{2}a_3)^{-\frac{1}{4}}\frac{a\mathrm{cn}\left(\frac{\sqrt{2s_1m_1(\alpha-\beta)}}{2mm_1}(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0),m\right)+b}{c\mathrm{cn}\left(\frac{\sqrt{2s_1m_1(\alpha-\beta)}}{2mm_1}(-\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0),m\right)+d}-\frac{a_2}{3a_3}. \quad (57)$$

Where

$$c=a-l_1-\frac{s_1}{m_1},d=a-l_1-s_1m_1,a=\frac{1}{2}[(\alpha+\beta)c-(\alpha-\beta)d],b=\frac{1}{2}[(\alpha+\beta)d-(\alpha-\beta)c],$$
$$E=\frac{s_1^2+(\alpha-l_1)(\beta-l_1)}{s_1(\alpha-\beta)},m_1=E\pm\sqrt{E^2+1},m^2=\frac{1}{1+m_1^2}. \quad (58)$$

We choose m_1 such that $\varepsilon m_1<0$.

Case 9. $D_4 > 0$, $D_2 D_3 \le 0$. Then we have

$$F(w) = [(w-l_1)^2 + s_1^2][(w-l_2)^2 + s_2^2]. \tag{59}$$

Where l_1, l_2, s_1, s_2 are real numbers, and $s_1 > s_2 > 0$.

When $\varepsilon = 1$, we have

$$w = \frac{a\,\mathrm{sn}\left(\eta(\xi-\xi_0), m\right) + b\,\mathrm{cn}\left(\eta(\xi-\xi_0), m\right)}{c\,\mathrm{sn}\left(\eta(\xi-\xi_0), m\right) + d\,\mathrm{cn}\left(\eta(\xi-\xi_0), m\right)}. \tag{60}$$

The corresponding solution is

$$u = (\frac{1}{2}a_3)^{-\frac{1}{4}} \frac{a\,\mathrm{sn}\left(\eta(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0), m\right) + b\,\mathrm{cn}\left(\eta(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0), m\right)}{c\,\mathrm{sn}\left(\eta(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0), m\right) + d\,\mathrm{cn}\left(\eta(\frac{1}{2}a_3)^{\frac{1}{4}}(\xi_1-\xi_0), m\right)} - \frac{a_2}{3a_3}. \tag{61}$$

Where

$$\begin{gathered} c = -s_1 - \frac{s_2}{m_1}, d = l_1 - l_2, a = l_1 c + s_1 d, b = l_1 d - s_1 c, \\ E = \frac{s_1^2 + s_2^2 + (l_1-l_2)^2}{2s_1 s_2}, m_1 = E + \sqrt{E^2-1}, m^2 = 1 - \frac{1}{m_1^2}, \eta = s_2\sqrt{\frac{m_1^2 c^2 + d^2}{c^2+d^2}}. \end{gathered} \tag{62}$$

In Eqs.(14)(17)(20)(22)(25)(27)(29)(31)(33)(35)(38)(40)(43)(46)(48)(50)(52)(55) (57) and (61), the integration constant ξ_0 has been rewritten, but we still use it. The classifications of all single traveling wave solutions to this equation are obtained.

CONCLUSION

In this paper, we use complete discrimination system for polynomial to solve the Gardner-KP equation. The advantage of this method is that we can deal with nonlinear equations with linear methods. This method has the characteristics of simple steps and effectivity. With the same method, some of other equations can be dealt with.

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