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**Research Article**

# Determination of Cost-Efficient Bandwidths of Multicast Algorithms through Data Envelopment Analysis using Variable Return to Scale Model

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**Abstract:** In this paper, Data Envelopment Analysis (DEA) was applied using Variable Return to Scale (VRS) model to determine bandwidth consumption of two algorithms during video conferencing over wireless network. Recent researches conducted focused on Constant Return to Scale (CRS) model where all Decision Making Units (DMUs) are operating at an optimal scale. DEA employing VRS was used on simulation results of Improved Network Coding Algorithms (INCAs) of two and three parameters to determine the better coding algorithm. The two Simulation results showed that, the INCA with three parameters is more efficient than the INCA with two parameters. This is ascertained from the mean efficiency of INCA with three parameters which are; 0.941, 0.934 and 0.812 for 20, 50 and 60 nodes generated at random when compared with mean efficiency of 0.847, 0.909 and 0.801 for INCA with two parameters using the same number of nodes generated at random.
**Keywords:** Variable Return to Scale, Data Envelopment Analysis, Efficiency, Decision Making Units and Video conferencing.

## INTRODUCTION

Data Envelopment Analysis (DEA) is a method of mathematical programming to measure the decision making units based on a series of observations, by which it can empirically estimate the efficiency frontier.DEA is one method that has been used widely for benchmarking since its introduction by Charnes, Cooper and Rhodes in 1978 [1]. Recent researches on the efficiency of multicast algorithms have gained significant attention, most especially, with the deployment of DEA as a tool for the measure of performance efficiency. DEA is a rapidly growing area of operational research that deals with the performance assessment of organizations or algorithms and can handle complex problems with multiple inputs and outputs. The development of computer based software for solving analytical linear programming problems has made it easier to use DEA in practical application [2]. Linear programming is the strength of DEA methodology that is based on optimization.

With the advent of 4G network, video conferencing over wireless network has become the fundamental means of group discussion over the internet. Prior to its implementation in real time situations, considerable amount of algorithm based-researches were conducted aimed at maximizing throughput, bandwidth utilization, evaluation of electric distribution utilities, energy consumption and performances [3-7]. The Improved Network Coding Algorithm (INCA) which was developed by [8] is an extension of the research work carried out by [9]. The application of DEA to determine the performance efficiency of multicast algorithms over wireless network is an open area of research that is receiving great attention in recent times.

Video conferencing is a communication technology that permits users at different locations around the world to interact by creating a face-to-face meeting environment. It transmits bi-directional audio, video and data streams during the whole session to all intending participants. These signals are compressed using software called Coders/Decoders (CODECs) and it is also used for convert between analog and digital formats. Equipment such as call server, video endpoint, Multipoint Conference Unit (MCU), gateways, and an Ethernet switch are mostly used during video conferencing. Figure 1 shows the architecture of multi-party video conferencing. The call server performs the registration and call control processing functions. Participants make and receive video calls from a device called video endpoints which process the bi-directional audio, video, data streams and interfaces to the participants.

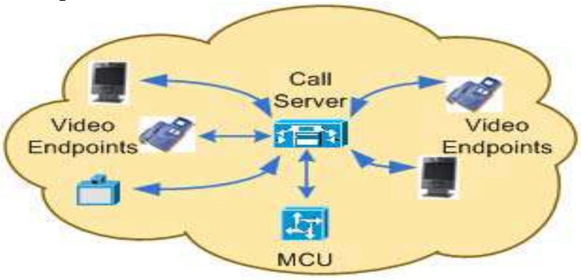


**Fig-1: Multi-Party Video Conferencing[10]**

Bandwidth consumption is the most crucial issue to focus on during video conferencing. The basic architecture of video conferencing is shown in Figure 1. This research, which is based on the performance efficiency of the INCA algorithms, is set to determine a better algorithm for such network architecture.

Decision Making Units (DMUs) is a model of efficiency analysis that uses multiple inputs for the production of multiple outputs.

The efficiency measure compares the ratio output/input of the DMU assessed with the value of this ratio observed in the other DMUs

The aim of this paper is to determine the performance efficiency of two multicast algorithms using DEA by employing Variable Return to Scale (VRS) model. Earlier researches focused on the determination of cost-effectiveness and cost efficiency using Constant Return to Scale (CRS) model. Research conducted by [8]determined the effectiveness of the INCA with two and three parameters in minimizing the consumption of bandwidth during multicast over wireless network. The research was aimed at reducing the consumption of bandwidth during multimedia application. Simulation results showed that the INCA with three parameter factors of delay, loss, and rejection of packets been addressed, largely minimized bandwidth consumption. However, the performance efficiency of the algorithms was not considered. Performance evaluation of electric distribution utilities based on DEA was carried out by [7]. They used DEA to address the performance analysis of the 50 largest sales electric distribution utilities in the USA based on Mega Watt hour (MWh). The simulation results obtained included performance efficiency, gaps in inputs and outputs of inefficient utilities, sensitivity-based classification of utilities, and gap report.

Determination of bandwidth efficient multicast using DEA was carried out by[11]. The researchers applied DEA using Constant Return to Scale (CRS) model to determine the performance efficiency of INCA with two and three performance parameters, respectively. Simulation results showed that the INCA with two parameters was the most cost-efficient algorithm. But the performance efficiency using the VRS was not considered.

Therefore, there is the need to fill in this gap by considering the performance efficiency of two multicast algorithms using Variable Return to Scale (VRS) model and hence, the motivation of this research.

**DEA Model**

The model used in this research focused only on input orientated DEA using Variable Return to Scale (VRS). Input oriented efficiency indicates by how much can input quantities be proportionally reduced holding output constant[12].

$$\min \theta$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} - y_{ij} \geq 0 \qquad (1)$$

$$\sum_{j=1}^{n} \lambda_j x_{kj} - s_n x_{kn} \leq 0 \qquad (2)$$

$$\sum_{j=1}^{N} \lambda_j = 1 \qquad (3)$$

$$\lambda_j \geq 0 \qquad (4)$$

where,
i=1, 2,…, I, k =1, 2, 3,…, K, j=1, 2, 3… n,

$\lambda_i$ is the weight given to $DMU_i$ in its efforts to dominate $DMU_0$ ,and $\theta$ is the efficiency of $DMU_0$ . $x_{1j}, x_{2j}, ..., x_{mj}$ are the m inputs and $y_{1j}, y_{2j}, ..., y_{sj}$ are the s outputs. $\lambda$'s and $\theta$ are the variables. Since $DMU_0$ appears on the left hand side of the equations as well, the optimal $\theta$ cannot be more than 1.

**Efficiency Measure using DEA**

One major component of performance is efficiency and it is defined as the ratio of outputs to inputs.

Therefore, if $x_{1j}, x_{2j}, ..., x_{mj}$ are the m inputs and $y_{1j}, y_{2j}, ..., y_{sj}$ are the s outputs of the unit j, then its efficiency, $\theta$, is defined as[6]:

$$\theta = \frac{u_1 y_{1j} + u_2 y_{2j} + ... u_s y_{si}}{v_1 x_{1j} + v_2 x_{2j} + ... v_m x_{mj}} \qquad (5)$$

where,
$v_1, v_2, \ldots, v_m$ are weights for the inputs and $u_1, u_2, \ldots, u_s$ are the weights for the outputs.

DEA concept can be graphically illustrated in Figure 3

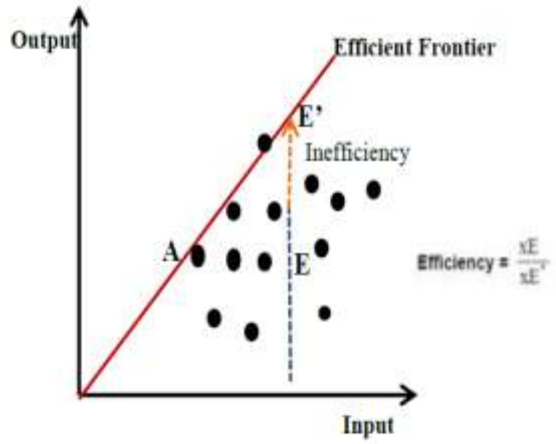


**Fig-2: DEA concept [13]**

Figure 2 shows single input and single output DEA using Constant Return to Scale Model.

Major efficiency concepts of DEA can be described as technical, scale, cost, and allocation efficiency. The efficiency concepts of DEA are clearly defined for proper understanding of how they are interpreted.

**Technical Efficiency**

The most common efficiency concept is technical efficiency which is the conversion of physical inputs into outputs relative to best practice. For example, consider bank A attending to customers on ATM cards issues. Bank A can provide 90 AMT cards to customers in 120 Minutes. In recent times, Bank A produced 70 ATM cards to customers in 120 minutes. The best achievable efficiency score for bank A is 0.75 (90/120), while due to their output of 70 AMT cards, their current efficiency score is 0.58 (70/120). Hence, Bank A is presently operating at 77.3% (0.58/0.75)efficiency only. This is called technical efficiency. In order for Bank A to become technically efficient, it would have to increase its current output by 20 customers per month[14].

**Scale efficiency:**

In scale efficiency, an algorithm can take advantage of returns to scale by altering its size towards optimal scale. This is defined as the region in which there are constant returns to scale in the relationship between outputs and inputs.

**Cost efficiency**

An algorithm is technically and allocation efficient if it produces a given quantity, quality, and combination of outputs at minimum possible cost by considering all the DMU's and parameters involved. Cost efficiency is the combination of technical and allocation efficiencies[15].

**Allocation efficiency**

Allocation efficiency refers to a situation where inputs for a given level of output and set of input prices are chosen to minimize the cost of production with the assumption that the organization being examined is already fully technically efficient. It is usually express in percentage[16].

In this paper, the main focus is on Cost Efficiency (CE) using VRS [17]. The application of DEA using VRS to networking multimedia applications in this work is a new development. Thus, efficiencies of different set of network configurations could be measures of their performance.

**SIMULATION RESULTS**

Simulation was carried out using DEA software Version 2.1. Input orientated DEA was used and the required input parameters are obtained from the simulation results of INCA with two and three parameters. The simulation parameters are saved in a data file named *sj6-dta.txt* and the instruction file named *sj6-ins.txt*. An output file was created with a name *sj6-out.txt*, which serves as a directory for saving the output results. Tables 1, 2 and 3 show the simulation results of DEA program. Column one represents the Decision Making Units (DMUs),column two and three show the simulation parameters results of cost effectiveness, while column four and five show results of cost efficiency for INCA with two and three performance parameters. The graphical representations of the simulation results for cost efficiency for 20, 50 and 60 randomly generated nodes in columns four and five of Tables 1, 2 and 3 are shown in Figures 2, 3 and 4, respectively.

**Table-1: Effectiveness and Efficiency of Bandwidth Consumption for 20 Random Nodes**

| 20 nodes | Cost-effectiveness | | Cost efficiency | |
|---|---|---|---|---|
| No. of participants | INCA with two metrics | INCA with three metrics | INCA with two metrics | INCA with three metrics |
| 2 | 0.3405 | 0.1571 | 1.000 | 1.000 |
| 3 | 1.5047 | 1.4453 | 0.356 | 0.856 |
| 4 | 1.6769 | 1.5297 | 0.571 | 1.000 |
| 5 | 2.2611 | 1.2306 | 1.000 | 1.000 |
| 6 | 3.2449 | 2.4076 | 0.999 | 0.890 |
| 7 | 4.9350 | 4.3894 | 0.816 | 0.818 |
| 8 | 5.5823 | 5.4192 | 0.878 | 0.912 |
| 9 | 6.2030 | 5.9324 | 1.000 | 0.990 |
| 10 | 8.4356 | 7.2994 | 1.000 | 1.000 |
| Mean | | | **0.847** | **0.941** |

**Table-2: Effectiveness and Efficiency of Bandwidth Consumption for 50 Random Nodes**

| 20 nodes | Cost-effectiveness | | Cost efficiency | |
|---|---|---|---|---|
| No. of participants | INCA with two metrics | INCA with three metrics | INCA with two metrics | INCA with three metrics |
| 2 | 0.7506 | 0.2555 | 1.000 | 1.000 |
| 3 | 1.6129 | 1.0598 | 0.571 | 0.652 |
| 4 | 2.0116 | 1.6066 | 1.000 | 0.967 |
| 5 | 2.8540 | 2.8184 | 1.000 | 1.000 |
| 6 | 4.3289 | 4.3147 | 0.890 | 1.000 |
| 7 | 5.9323 | 5.0145 | 0.818 | 0.857 |
| 8 | 6.4192 | 6.2312 | 0.912 | 0.935 |
| 9 | 6.9220 | 6.0914 | 0.990 | 1.000 |
| 10 | 7.8556 | 7.0058 | 1.000 | 1.000 |
| Mean | | | **0.909** | **0.934** |

**Table-3: Effectiveness and Efficiency of Bandwidth Consumption for 60 Random Nodes**

| 20 nodes | Cost-effectiveness | | Cost efficiency | |
|---|---|---|---|---|
| No. of participants | INCA with two metrics | INCA with three metrics | INCA with two metrics | INCA with three metrics |
| 2 | 1.8676 | 1.0772 | 1.000 | 0.793 |
| 3 | 2.5694 | 1.1341 | 0.445 | 0.577 |
| 4 | 1.4818 | 1.2087 | 1.000 | 1.000 |
| 5 | 4.7344 | 4.2347 | 0.436 | 0.475 |
| 6 | 4.2993 | 4.1223 | 0.603 | 0.702 |
| 7 | 3.7835 | 3.1230 | 1.000 | 1.000 |
| 8 | 5.9203 | 5.8159 | 0.726 | 0.819 |
| 9 | 6.2568 | 5.3186 | 1.000 | 0.946 |
| 10 | 6.9860 | 6.5559 | 1.000 | 1.000 |
| Mean | | | **0.801** | **0.812** |

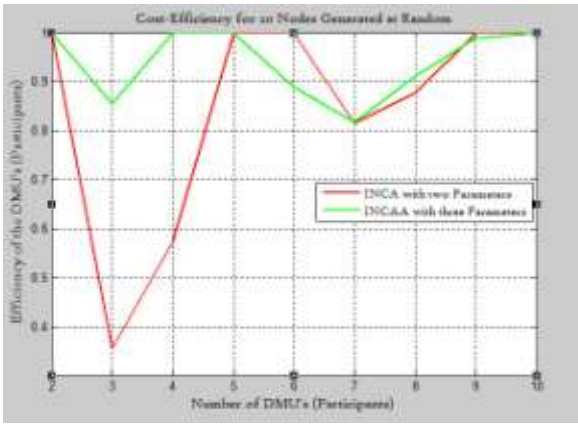


**Fig-2: Cost-Efficiency of Multicast Algorithm for 20 Nodes Generated at Random**

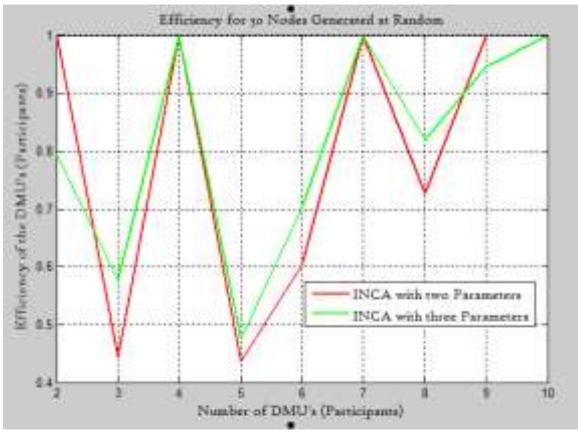


**Fig-3: Cost-Efficiency of Multicast Algorithm for 50 Nodes Generated at Random**

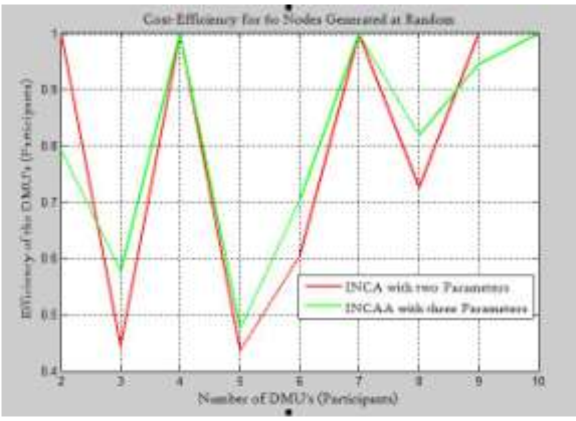


**Fig-4: Cost-Efficiency of Multicast Algorithm for 60 Nodes Generated at Random**

## RESULTS AND DISCUSSION

It is evident from the simulation results obtained in Tables 1, 2, and 3 that the INCA with three parameters is the most cost efficient in terms of performance when compared with the INCA with two parameters. The findings from graphs also show that the INCA with three parameters outperformed the INCA with two parameters. For example, the mean efficiency of the INCA with three parameters for 20, 50, and 60 nodes generated at random is 0.941, 0.934 and 0.812 when compared with 0.847, 0.909 and 0.801 obtained from INCA with two parameters.

Measuring the performance of a network using only effectiveness as was done by most researchers is not adequate, but an additional examination of its efficiency too will give better and true performance picture of the network. Hence, this was the reason and attention in this research by introducing DEA to evaluate the cost-efficiency of multicast algorithms (INCA) during video conferencing over a wireless network. With the growing demand in multimedia application nowadays, Video Conferencing is becoming the most efficient method of supporting group communication over wireless network.

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