

Measuring the Efficiency of the Dairy Industry: Using DEA Models

Jin Ee Mo, Zi Yi Mok*, Slyvia Moh Sze Tan, Dariush Khezrimotlagh

Department of Applied Statistics, Faculty of Economics and Administration, University of Malaya, 50603 Kuala Lumpur, Malaysia

*Corresponding Author

Zi Yi Mok

Email: ziyi.mok@siswa.um.edu.my

Abstract: This paper measures the relative efficiency of some of milk-producing states in the United States (US) by Data Envelopment Analysis (DEA). 23 milk production states are selected with 12 factors, inclusive 8 inputs and 4 outputs. The recent unique model, Kourosh and Arash Model (KAM), is used to discriminate, rank and benchmark the states. There are 20 technically efficient states while Constant Returns to Scale (CRS) is applied. However, KAM suggests there is only one efficient company when thickness of the DEA frontier is less than 0.017 and other companies are inefficient with one millionth degree of freedom in inputs and outputs. Indeed, if only one millionth errors of the minimum values of positive factors are introduced in inputs and outputs values of the DEA frontier, the efficiency scores of technically efficient companies are sharply decreased. Reference sets for each technically efficient company as well as the rank of all companies are illustrated. Simulations are also calculated with Microsoft Excel Solver 2013.

Keywords: Data envelopment analysis, Kourosh and Arash model, Efficiency, Dairy industry

INTRODUCTION

The dairy industry is labor intensive and provides many job opportunities. It is essential in contributing to the economy by providing various job opportunities at all levels inclusive of manufacturing, transportation, marketing and so on. In 2012, the USA was the largest cow milk producer in the world, it accounted for 14.6% of the world production. Its dairy farms produce roughly 23 billion gallons of milk annually, and are estimated at \$140 billion in economic output, \$29 billion in household earnings which create more than 900,000 jobs [1].

In order to find the most efficient milk production states in the USA in 2012, this study considers 23 milk productions with 12 factors, inclusive 8 inputs such as purchased feed, milking frequency, hired labour and etc. and 4 outputs such as milk sold, output per cow and etc. Data were acquired from the United States Department of Agriculture-Economic Research Service [2].

Data Envelopment Analysis (DEA) is applied to discriminate the selected states. DEA is a nonparametric method to measure the relative efficiency of a set of homogenous Decision Making Units (DMUs). It was first introduced by Charnes, Cooper and Rhodes (CCR) in 1978 [3], and has dramatically been improved in the last three decades. Recently a robust model, called Kourosh and Arash Model (KAM) [4] was

proposed which uniquely improves the discrimination power of DEA, and depicts DEA as the most robust technique among current parametric and non-parametric methodologies.

KAM provides an efficiency score for each DMU as well as simultaneously benchmark and rank DMUs. It classifies DMUs into three different sets, named efficient, technically efficient and inefficient DMUs while an epsilon Degree of Freedom (DF) is introduced in the DEA frontier. The rest of this paper is organized into 5 sections. The next section is a short background on milk production and introducing KAM. In Section 3 data and how to apply KAM are illustrated. The results of KAM are represented in Section 4 and the paper is concluded in Section 5.

LITERATURE REVIEW

There are various factors that affect milk production. Bar-peled et al. [5] found that cows that were suckled has highest milk production, followed by cows that were milked six times daily, while cows that were milked three times daily have the lowest milk production. In another study, it was discovered that cows that were milked 3 times daily increases milk yield by 18% compared to cows that were milked twice daily [6].

Machlin [7] stated that milk production increased by 18% when the cows were injected with the Bovine

growth hormones daily for 10 days. Bewley et al. [6] also discovered that the use of bST increases milk production by 1286 lb in the 1999 Wisconsin Dairy Modernization Project. bST is claimed to help increase 9 to 13 pound or 10 to 15% increase in milk production per cow per day [8]. Tauer [9] mentioned that efficient small dairy farms can be competitive with larger dairy farms in New York in producing milk at comparable costs per unit [9].

Oleggini et al. [10] discovered that larger herds have higher Income Over Feed Cost (IOFC) than smaller herds, and thus are more cost efficient. In the United States of America, it was established that milk production was the highest in the Northern region, followed by Mid-south. The Southern region has the lowest milk production.

Silva et al. [11] measured the technical efficiency of Azores dairy farms in Portugal using DEA with Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS). They claimed that there are efficient and inefficient farms, regardless of their farm size and technology advancements. Stokes et al. [12] performed CCR DEA on Pennsylvania dairy farms and found that too much labor use, overinvestment in land in conjunction with too little milk and butterfat production, and overuse of debt capital are the causes of inefficient dairy production. Kelly et al. [13] also performed DEA on Irish dairy farms, under both assumptions of CRS and VRS, and found that efficient producers had higher production per cow and per hectare with a longer grazing season, better milk quality, were more likely to have participated in milk recording and had superior land quality.

DEA is a well-known non-parametric method in operations research which by a simple linear programming discriminates between a set of homogenous DMUs and measures their relative efficiency scores. It first defines a Production Possibility Set (PPS) and considers its frontier as the best available estimation of the production frontier.

A DMU which does not lie on the frontier is called an inefficient DMU, and is called technically efficient if it lies on the frontier. Conventional DEA models benchmark inefficient DMUs toward the frontier, but are not able to benchmark technically efficient DMUs toward the best economically part of the frontier [4].

A recent robust model in DEA, called KAM, was proposed by Khezrimotlagh et al. [15, 16] to increase the discrimination power of DEA uniquely. KAM introduces a very negligible thickness in the frontier and benchmark technically efficient DMUs as well as inefficient DMUs toward the economically part of the

frontier. The ε -KAM in CRS for n DMUs with m inputs and p outputs is given by:

$$\begin{aligned} & \max \sum_{j=1}^m w_j^- s_j^- + \sum_{k=1}^p w_k^+ s_k^+ \\ & \text{Subject to} \\ & \sum_{i=1}^n \lambda_i x_{ij} + s_j^- = x_{ij} + \varepsilon_j^-, \text{ for } j = 1, 2, \dots, m, \\ & \sum_{i=1}^n \lambda_i y_{ik} - s_k^+ = y_{ik} + \varepsilon_k^+, \text{ for } k = 1, 2, \dots, p, \\ & x_{ij} - s_j^- \geq 0, \text{ for } j = 1, 2, \dots, m, \\ & y_{ik} + s_k^+ - 2\varepsilon_k^+ \geq 0, \text{ for } k = 1, 2, \dots, p, \\ & \lambda_i \geq 0, \text{ for } i = 1, 2, \dots, n, \\ & s_j^- \geq 0, \text{ for } j = 1, 2, \dots, m, \\ & s_k^+ \geq 0, \text{ for } k = 1, 2, \dots, p. \end{aligned}$$

The KAM best technical efficiency score and target with ε degree of freedom (ε -DF) are as follows:

$$\begin{aligned} x_{ij}^* &= x_{ij} - s_{ij}^- + \varepsilon_j^-, \text{ for } j = 1, 2, \dots, m, \\ y_{ik}^* &= y_{ik} + s_{ik}^+ - \varepsilon_k^+, \text{ for } k = 1, 2, \dots, p, \end{aligned}$$

$$KA_{\varepsilon}^{*l} = \frac{\sum_{k=1}^p w_k^+ y_{lk} / \sum_{j=1}^m w_j^- x_{lj}}{\sum_{k=1}^p w_k^+ y_{lk}^* / \sum_{j=1}^m w_j^- x_{lj}^*}.$$

If the value of epsilon is 0, KAM is the same as the weighted Additive DEA model (ADD) suggested by Charnes et al. [3], and is almost completely the same as the non-linear model Slack-Based Measure (SBM) suggested by Tone [14].

Data Selection

The used data are illustrated in Table 1, gathered from the United States Department of Agriculture-Economic Research Service [2]. It consists of milk production costs and returns per hundredweight sold by the States in the USA in 2013. The relevant input data are as follows:

- x_1 : Purchased feed: cost of purchase of grain/concentrates, fodder and other supplements to feed the cows,
- x_2 : Veterinary and medicine: cost of treatments and medication,
- x_3 : Bedding and litter: cost of bedding and litter for the cows,
- x_4 : Marketing: cost of marketing for the farm,
- x_5 : Hired labor: cost of hiring labor for the farm,
- x_6 : General farm overhead: cost of overhead for the farm,
- x_7 : Milking frequency more than twice per day (percent of farms): percentage of farms with frequency of milking more than twice a day out of total farms in the respective states,
- x_8 : Milk cows receiving bST (percent of cows): percentage of milk cows receiving hormone injections,

The relevant output data are given by:

y_1 : Milk sold: amount of milk sold,
 y_2 : Cattle: number of cattle in the farm,
 y_3 : Output per cow (pounds): milk output in pounds for the cow,

y_4 : Organic milk sold (percent of sales): percentage of milk sales that is organic out of total milk sales,

Table 2 illustrates the 23 milk producers and the selected corresponding codes.

Table 1: The 23 DMUs with 8 Inputs and 4 Outputs.

DMUs	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	y_1	y_2	y_3	y_4
A01	11.15	0.60	0.08	0.29	1.53	0.43	17.42	5.06	17.92	1.69	22135	1.32
A02	12.12	0.60	0.01	0.16	2.68	0.55	18.01	0.00	23.59	2.03	18101	0.00
A03	9.21	0.62	0.02	0.21	1.99	0.43	3.03	0.00	22.35	1.31	15594	0.00
A04	8.59	0.58	0.25	0.27	1.47	0.28	23.93	0.00	19.52	1.37	22059	2.10
A05	6.78	1.01	0.34	0.19	1.50	0.50	0.00	3.25	19.71	1.69	17879	1.46
A06	9.81	1.05	0.28	0.15	0.98	0.62	6.07	0.00	21.32	2.04	19243	2.77
A07	8.58	0.95	0.44	0.16	1.50	0.62	6.05	13.16	19.64	2.17	21113	2.63
A08	8.83	1.49	0.15	0.23	1.35	0.32	16.79	32.53	19.64	1.81	21153	0.00
A09	8.41	0.72	0.26	0.32	1.56	0.50	2.95	1.52	25.32	1.96	14376	2.89
A10	14.25	1.11	0.91	0.31	2.54	0.96	0.00	0.00	23.41	1.36	17647	8.33
A11	9.13	0.76	0.31	0.20	1.83	0.62	13.97	0.00	20.26	1.33	22206	0.38
A12	6.91	0.89	0.42	0.20	1.08	0.67	8.18	18.89	20.03	2.02	20342	2.99
A13	10.27	0.70	0.11	0.15	0.93	0.63	0.00	0.58	19.78	2.04	14135	1.00
A14	9.72	1.15	0.44	0.29	1.57	0.79	11.72	13.77	21.32	1.11	19686	4.18
A15	9.12	0.83	0.51	0.21	1.21	0.64	6.92	1.09	21.40	2.17	21025	3.73
A16	14.55	0.89	0.22	0.16	2.83	0.43	6.47	0.00	24.67	1.87	19129	26.88
A17	9.67	1.05	0.43	0.22	1.03	1.00	7.92	20.67	21.74	1.45	19729	2.82
A18	12.13	0.82	0.09	0.26	2.18	0.61	2.90	0.00	23.15	1.41	14391	0.00
A19	11.57	0.50	0.04	0.13	1.62	0.27	9.51	1.11	18.78	1.67	16753	0.53
A20	11.26	0.80	0.47	0.37	1.30	1.00	3.29	0.85	22.40	1.26	17471	8.49
A21	10.43	0.97	0.27	0.16	1.69	0.93	10.05	0.00	21.95	1.66	20200	1.02
A22	12.20	0.87	0.25	0.28	2.15	0.67	20.96	0.00	20.82	1.20	20272	6.07
A23	6.19	1.03	0.34	0.23	1.77	0.80	11.25	22.07	20.51	1.61	20133	3.84

Table 2: The 23 Milk Producers and Corresponding Codes.

State	Code	State	Code	State	Code	State	Code
California	A01	Iowa	A07	Missouri	A13	Texas	A19
Florida	A02	Kansas	A08	York	A14	Vermont	A20
Georgia	A03	Kentucky	A09	Ohio	A15	Virginia	A21
Idaho	A04	Maine	A10	Oregon	A16	Washington	A22
Illinois	A05	Michigan	A11	Pennsylvania	A17	Wisconsin	A23
Indiana	A06	Minnesota	A12	Tennessee	A18		

Applying KAM and its Results

In order to apply KAM, since the weights are unknown, they are defined as $w_j^- = 1/x_{ij}$ and $w_k^+ = 1/y_{lk}$, where $x_{ij} > 0$ and $y_{lk} > 0$, and if $x_{ij} = 0$ or $y_{lk} = 0$, the weights are defined as 1.

Since the only KAM best technical efficiency scores and targets are interested in this study, according to

Khezrimotlagh et al. [4], the following constraints are removed from KAM:

$$x_{ij} - s_j^- \geq 0, \text{ for } j = 1, 2, \dots, m,$$

$$y_{lk} + s_{lk}^+ - 2\varepsilon_k^+ \geq 0, \text{ for } k = 1, 2, \dots, p.$$

The components of epsilon vector, ε_j^- and ε_k^+ , are defined as $\varepsilon \times \min\{x_{ij}: x_{ij} \neq 0, i = 1, 2, \dots, n\}$ and

$\varepsilon \times \min\{y_{ik} : y_{ik} \neq 0, i = 1, 2, \dots, n\}$, respectively, while ε is selected as $0.000001 = 10^{-6}$. Indeed, the

minimum values of inputs and outputs are:

Factors	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	y_1	y_2	y_3	y_4
Min Values	6.19	0.5	0.01	0.13	0.93	0.27	2.9	0.58	17.92	1.11	14135	0.38

Therefore, the components of epsilon vector are $\varepsilon_1^- = 0.00000619$, $\varepsilon_2^- = 0.00000050$, $\varepsilon_3^- = 0.00000001$, $\varepsilon_4^- = 0.00000013$, $\varepsilon_5^- = 0.00000093$, $\varepsilon_6^- = 0.00000027$, $\varepsilon_7^- = 0.00000290$, $\varepsilon_8^- = 0.00000058$, $\varepsilon_1^+ = 0.00001792$, $\varepsilon_2^+ = 0.00000111$, $\varepsilon_3^+ = 0.01413500$ and $\varepsilon_4^+ = 0.00000038$, which are completely negligible according to minimum values of each factor.

As it is illustrated in the next section, KAM with this very small negligible thickness of the frontier, simultaneously arranges and benchmarks all technically efficient and inefficient DMUs appropriately. After applying KAM for each evaluated DMU, the non-zero optimum values of λ_i introduce the reference sets of the

DMU by 10^{-6} -DF. Moreover, according to Khezrimotlagh et al. [4] the value of δ is introduced as $\varepsilon/(m + p)$, that is, $10^{-6}/12$ to get decision whether DMU is efficient with 10^{-6} -DF or inefficient with 10^{-6} -DF. Indeed, if the value of $KA_0^* - KA_{10^{-6}}^*$ is less than $10^{-6}/12$, the DMU is called efficient with 10^{-6} -DF, otherwise, it is called inefficient with 10^{-6} -DF. It is also possible to introduce δ as $\varepsilon/10$, that is, $\delta = 10^{-7}$, to increase the number of efficient DMUs with 10^{-6} -DF. Table 4 illustrates the results of Charnes, Cooper and Rhodes (CCR) model in Input Oriented (IO) [3], as well as ADD in CRS (that is 0-KAM) and 10^{-6} -KAM CRS.

Table 3: The results of applying DEA models.

DMU	CCR-IO	0-KAM	10^{-6} -KAM	Rank	Reference Sets	Decision
A01	1.0000	1.0000	0.99999687	11	A01, A16, A19.	Inefficient with 10^{-6} -DF
A02	1.0000	1.0000	0.99999974	3	A02, A16.	Inefficient with 10^{-6} -DF
A03	1.0000	1.0000	0.99999914	5	A03, A16.	Inefficient with 10^{-6} -DF
A04	1.0000	1.0000	0.99999618	15	A04, A06, A16.	Inefficient with 10^{-6} -DF
A05	1.0000	1.0000	0.99999790	9	A05, A13, A16.	Inefficient with 10^{-6} -DF
A06	1.0000	1.0000	0.99999834	7	A06, A13, A16.	Inefficient with 10^{-6} -DF
A07	1.0000	1.0000	0.99999626	14	A05, A07, A12, A15, A16.	Inefficient with 10^{-6} -DF
A08	1.0000	1.0000	0.99998847	17	A01, A04, A06, A08, A16.	Inefficient with 10^{-6} -DF
A09	1.0000	1.0000	0.99999685	12	A09, A13, A16.	Inefficient with 10^{-6} -DF
A10	1.0000	1.0000	0.99999942	4	A10, A13, A16.	Inefficient with 10^{-6} -DF
A11	1.0000	1.0000	0.99996187	19	A04, A11, A15, A16.	Inefficient with 10^{-6} -DF
A12	1.0000	1.0000	0.99999857	6	A12, A15, A16.	Inefficient with 10^{-6} -DF
A13	1.0000	1.0000	0.99999724	10	A13, A16.	Inefficient with 10^{-6} -DF
A14	0.8280	0.5796	0.57955638	23	A05, A15, A16.	Inefficient
A15	1.0000	1.0000	0.99999827	8	A06, A13, A15, A16.	Inefficient with 10^{-6} -DF
A16	1.0000	1.0000	1.00000000	1	A16.	Efficient with 10^{-6} -DF
A17	0.9971	0.7699	0.76992402	21	A06, A09, A12, A16.	Inefficient
A18	1.0000	1.0000	0.99996798	18	A03, A10, A13, A16, A18.	Inefficient with 10^{-6} -DF
A19	1.0000	1.0000	0.99999376	16	A02, A16, A19.	Inefficient with 10^{-6} -DF
A20	1.0000	1.0000	0.99999644	13	A06, A13, A16, A20.	Inefficient with 10^{-6} -DF
A21	1.0000	1.0000	0.99993535	20	A02, A06, A11, A16, A19, A21	Inefficient with 10^{-6} -DF
A22	0.8247	0.5848	0.58483928	22	A04, A06, A16.	Inefficient
A23	1.0000	1.0000	0.99999991	2	A05, A12, A16, A23.	Inefficient with 10^{-6} -DF

As shown in Table 3, A14, A17 and A22 are inefficient. Indeed, CCR-IO and ADD CRS (0-KAM CRS) scores for these DMUs are less than 1, which are represented in columns 2 and 3. There are 20 technically efficient DMUs with the technical efficiency scores as 1 by CCR-IO and ADD CRS models.

Columns 4-6 of Table 3 demonstrate the best technical efficiency scores, ranks and reference sets of each DMU with 10^{-6} -DF. From the last column of the table, if one millionth errors of the minimum values of positive factors in inputs and outputs values of the frontier are introduced, by selecting δ as $10^{-6}/(8+4)$, only A16 (Oregon) is efficient with 10^{-6} -DF and other

technically efficient DMUs are inefficient with 10^{-6} -DF. In other words, if a small negligible thickness of the frontier is introduced in 12 dimensions space of inputs and outputs, the best technically efficient DMU is A16. From the table A16 is a reference set for all DMU, too.

The value of δ was also considered as 10^{-7} . Although, in this case, A16 and A23 are efficient with 10^{-6} -DF, A16 is a reference set for A23 which again shows A16 as the best technically efficient DMU among all DMUs. The robustness of KAM in comparison with other models is clearly seen in this practice by comparing the columns 2 and 3 with columns 4 -7.

CONCLUSION

This study measures the relative efficiency of milk production of 23 states of the USA. Three states such as York, Pennsylvania and Washington are found technically inefficient. Other states are technically efficient, however, they are inefficient with 10^{-6} -DF except Oregon emerged as the only efficient state with 10^{-6} -DF. The study shows KAM as a robust technique to rank and benchmark DMUs.

REFERENCES

1. Dairy. (N.D.). Retrieved October 27, 2014, from <http://en.wikipedia.org/wiki/Dairy>.
2. Dairy Management Inc. (DMI), National Milk Producers Federation (NMPF), United States Department of Agriculture (USDA), and National Agricultural Statistics Services (NASS).
3. Charnes A, Cooper WW, Golany B, Seiford LM, Stutz J; Foundations of data envelopment analysis and Pareto-Koopmans empirical production functions. *Journal of Econometrics*, 1985; 30(1): 91-107.
4. Khezrimotlagh D, Salleh S, Mohsenpour Z; A New Method for Evaluating Decision Making Units in DEA. *Journal of the Operational Research Society*, 2013; 65(5), 694-707.
5. Bar-peled U, Maltz E, Bruckental I, Folman Y, Kali Y, Gacitua H, Lehrer AR; Relationship between Frequent Milking or Suckling in Early Lactation and Milk Production of High Producing Dairy Cows. *Journal of Dairy Science*, 1994; 78:2726.
6. Bewley J, Palmer RW, Jackson-Smith DB; Modeling Milk Production and Labor Efficiency in Modernized Wisconsin Dairy Herds. *Journal of Dairy Science*, 2001; 84:705.
7. Machlin LJ; Effect of Growth Hormones on Milk Production and Feed Utilization in Dairy Cows. *Journal of Dairy Science*, 1973; 56:575.
8. Schutz M, Kenyon S; BST in milk. In: *Purdue animal issues briefings* [on line]. Disponível na internet via [www. URL:](http://www.avr.les.purdue.edu/anissue/ait.html)
9. Tauer LW; Efficiency and Competitiveness of the Small New York Dairy Farm. *Journal of Dairy Science*, 2001; 84:2573.
10. Oleggini GH, Ely LO, Smith JW; Effect of Region and Herd Size on Dairy Herd Performance Parameters. *Journal of Dairy Science*, 2001; 84:1044.
11. Silva E, Arzubi A, Berbel J; An Application of Data Envelopment Analysis in Azores Dairy Farms. *New Medit* 2004; 3:2004.
12. Stokes J, Tozer P, Hyde J; Identifying efficient dairy producers using data envelopment analysis. *Journal of Dairy Science*, 2006; 90:2555.
13. Kelly E, Shalloo L, Geary U, Kinsella A, Wallace M; Application of data envelopment analysis to measure technical efficiency on a sample of Irish dairy farms. *Irish Journal of Agricultural and Food Research*, 2012; 51: 63-77.
14. Tone K; A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research*, 2001; 130(3): 498-509.
15. Khezrimotlagh D, Salleh S, Mohsenpour Z; Nonlinear Arash Model in DEA, *Research Journal of Applied Sciences, Engineering and Technology*, 2013; 5(17): 4268-4273.
16. Khezrimotlagh D; How to Select an Epsilon in Kourosh and Arash Model, Emrouznejad, A., R. Banker R., S. M. Doraisamy and B. Arabi, 2014, *Recent Developments in Data Envelopment Analysis and its Applications*, Proceedings of the 12th International Conference of DEA, April 2014, Kuala Lumpur, Malaysia, 2014.