

## Optimization of (R, Q) policy for two-echelon inventory system with lost sales through markov chain

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**Abstract:** This paper discusses a two-echelon inventory system consisting of one warehouse and two retailers. All installations use (R, Q) policy to replenish their stock. The transmission time between any two echelons follows the exponential distribution. A mathematical model is formulated to determine the optimal values of R and Q in both echelons, so the total cost is minimized. The total cost includes the average inventory cost, the average lost sales cost and the average ordering cost. Markov chain is used to model this system. The total cost function obtained is the type of a nonlinear integer programming and a genetic algorithm is proposed to solve it. Finally a numerical example is used to illustrate the performance of the model.

**Keywords:** Two-echelon inventory system; Markov chain; Lost sales; Batch ordering; Genetic algorithm.

### INTRODUCTION

Successful inventory management is one of the most important activities in numerous businesses. It can do many things for businesses, but above all it increases operational efficiency, improves customer service, and reduces inventory costs of supply network. One of the most special aspects of the inventory management that has crucial role in supply chain management is distribution of goods in multi-echelon inventory systems. Academicians and practitioners are widely using Multi-echelon inventory systems in many industries and communication networks. One of the earliest researches on multi-level inventory to manage the repairable spare parts, by Sherbrooke[1] presented METRIC as a suitable method to determine the optimal level of inventory in the inventory system. It was a two-level inventory system consisting of a central base and several local bases with an Inventory control policy of (S-1, S), continuous review for all facilities. Axsäter[2] developed a simple solution procedure for a two-echelon inventory system with one-for-one replenishment, constant lead-time, and independent Poisson demand at retailers. He applied an inventory cost function and concentrated on evaluating the average costs. Nahmias and Smith [3] presented a two-echelon inventory system with a distribution center and a number of retailers. All installations applied order up to S policy. The demand in retailers had a negative binomial distribution and a fraction of unsatisfied demands in the retailers were lost. Forsberg [4] considered a two-level inventory system with one warehouse and N retailers. In this model retailers were facing different compound Poisson demand processes and all facilities applied orders up to S replenishment policies and show how to evaluate holding and shortage costs exactly. Forsberg [5] developed his model in a way that all facilities use (R,Q) policy and presented the cost of the policy as a weighted average of costs for one-for-one ordering policies. Axsäter [6] developed his previous model with two retailers, and calculated the approximate cost for the system. Axsäter and Zhang[7] considered a two-level inventory system with a central warehouse and a number of identical retailers. When the sum of the retailers inventory positions have been declined to a certain joint reorder point, the retailer with the lowest inventory position placed an order. Furthermore, Seo et al., [8] presented an optimal reorder policy to utilize centralized stock information for a two-echelon inventory system consisting of one warehouse and multiple retailers that was controlled by continuous review batch ordering policy. Seifbarghy and Jokar,[9] investigated a two-echelon inventory system consisting of a warehouse, many identical retailers with lost sales and independent Poisson demands. They developed an approximate cost function to find the optimal reorder points for a given batch size. Haji et al., [10] introduced a new ordering policy for inventory control in a two-echelon inventory system consisting of one central warehouse and a number of non-identical retailers and showed how the inventory costs can be determined for this system. The warehouse was using a modified one-for-one policy and each retailer constantly placed an order for one unit of product to the central warehouse in a pre-determined time interval. Pasandideh et al., [11] considered a two-echelon inventory system for non-

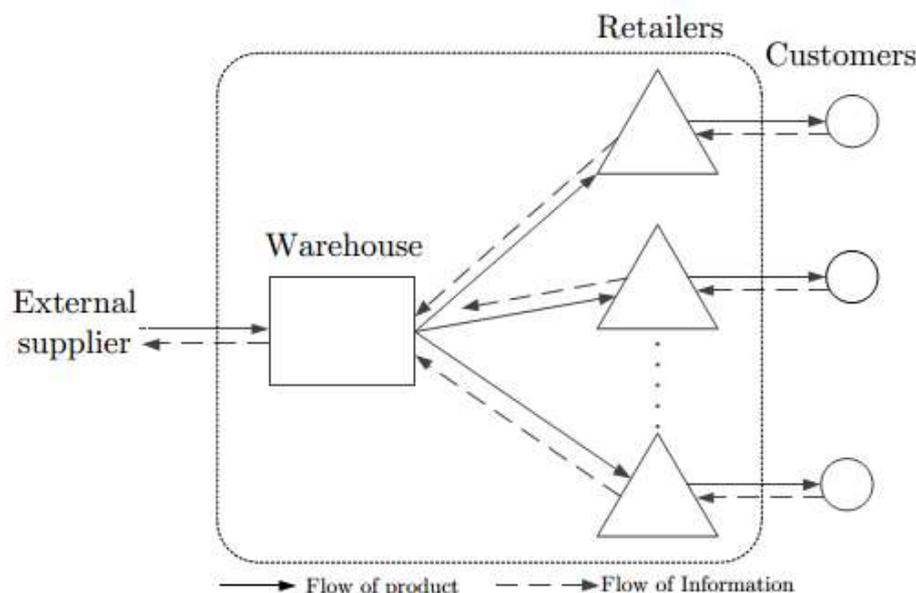
repairable item where the system was consist of one warehouse, many identical retailers and continuous-review (R,Q) ordering policy. They tried to find an effective stocking policy for this system by minimizing the total annual inventory investment. Seifbarghy et al., [12] presented his model consisting of one central warehouse and a few nonidentical retailers controlled by a continuous review inventory policy (R,Q).They estimated the cost function of the system utilizing a Response Surface Method (RMS) and obtained the optimal reorder points for given batch sizes. moreover, Ghiami et al., [13] also modeled a two-echelon supply chain system ,consist of a retailer with limited capacity and one wholesaler ,and aimed his study at minimizing the total cost of the system. Tai and Ching, [14] considered a Markovian model for two-echelon inventory/ return system and aimed at minimizing the total expected operating cost by choosing the maximum inventory level at the local warehouse.

In this paper, we consider a two-echelon inventory system consisting of a warehouse and two retailers. Unlike most researches that are considering the one for one policy, here we are using (R,Q) policy. Also the modeling is done in Markov chain. By using the assumptions and the method of modeling, compared to earlier studies, this paper provides a new way of considering the two-echelon inventory system.

**PROBLEM FORMULATION**

Consider a two-echelon inventory system consisting of a warehouse and two non-identical retailers. The system is assumed to work in the following manner:

At the retailer level, when a customer refers to the retailer, if the amount of on-hand inventory is positive, the demand is immediately satisfied, but if the amount of on-hand inventory is zero, the customer demand is lost. We assume that the service time and also the transfer time is negligible or zero compared to arrival of two consecutive customers. At warehouse level, when the warehouse is facing orders of retailers, if the inventory position is positive, the order is placed immediately; otherwise, the order is backordered. Service time to orders of retailers by warehouse is assumed negligible or zero. At both echelons, (R,Q) ordering policy continuously monitors the inventory position for each item. It means that as soon as the stock level declines to the reorder point R, an order of batch size Q is placed. The pictorial representation of the system under study is depicted in Fig-1.



**Fig-1:Two-echelon inventory system**

In order to define the problem precisely the following assumptions must be made:

1. Process of demand in the retailer  $i$  follows a Poisson distribution with the rate of  $\lambda_i$ .
2. Customers demand one unit from one type of product.
3. Lateral transfers between two retailers are not allowed.
4. Batch sizes of all retailers are the same.

According to the inventory review policy (R,Q), if the amount of on-hand inventory at the retail level is less than or equal to R, an order replenishment is placed on the warehouse. Similarly, if the inventory position in the warehouse is less than or equal to R, order replenishment is placed on the external Supplier with infinite capacity. After receiving the orders, all backordered orders are filled according to a FIFO-policy. Transfer time from the warehouse to retailer  $i$  has an exponential distribution with mean  $1/\mu_i, (i=1, 2)$ , and transfer time from an external supplier to warehouse also has an exponential distribution with mean  $1/\mu_0$ . Similar to what Axsäter and Zhang, [7] and Seifbarghy and Jokar[9] proposed in their researches, the reorder point and batch size of the warehouse are assumed to be integer multiple of the retailer's batch size. As per this assumption, orders will not be satisfied partially. In all installations,  $R \leq Q$  is assumed; with this assumption, there will be only one on order inventory between each echelon. Thus, the maximum inventory position at all installation will be an  $R + Q$ , and minimum inventory position at the warehouse will be  $-(\sum Q_i)$ . One of the natural limitations of the real world is a certain capacity of product transportation between different echelons. So, the echelons should consider transition capacity as a parameter for optimizing the batch size of the order.

The following notations, parameters and variables are used for mathematical formulation of the model:

$N$	Number of retailers $i = 1, 2$
$\lambda_i$	Demand rate at retailer $i$
$\mu_i$	Retailer $i$ replenishment rate
$\mu_0$	Warehouse replenishment rate
$Q_i$	Batch size at retailer $i$
$Q_0$	Batch size at warehouse
$R_i$	Reorder point at retailer $i$
$R_0$	Reorder point at warehouse
$h_i$	Holding cost per unit per unit time at retailer $i$
$h_0$	Holding cost per unit per unit time at warehouse
$l_i$	Opportunity cost of losing a customer at retailer $i$
$r_i$	Fixed order cost at retailer $i$
$r_0$	Fixed order cost at warehouse
$n$	On-hand inventory at the retailer $i$
$m$	Inventory position at warehouse
$\pi$	Steady-state probability

Let  $m(t)$  denote the inventory position (inventory on hand minus backlog) at the warehouse at time  $t \geq 0$ ,  $n_1(t)$  denote the on hand inventory at the first retailer at time  $t \geq 0$ , and  $n_2(t)$  denote the on hand inventory at the second retailer at time  $t \geq 0$ . Now we can define the state of the system as  $Z=(m(t), n_1(t), n_2(t), t \geq 0)$  and the state space of  $Z$  as follows:

$$E_Z = \{(m, n_1, n_2) : -(\sum_{i=1}^2 Q_i) \leq m \leq R_0 + Q_0, \\ 0 \leq n_1 \leq R_1 + Q_1, 0 \leq n_2 \leq R_2 + Q_2\} \quad (1)$$

According to the mentioned assumptions and state of the system, it is obvious that sojourn times in each state are exponentially distributed; hence  $Z$  is a continuous-time Markov chain. Some states of the set  $E_z$  are always zero, because they may not occur. These impossible states are presented through operators  $(A_i, C_i$  and  $F_i)$  with values equal to zero. Similar to what Saffari et al. [15] and Chiang and Monahan [16] did in their researches the equilibrium equations of all states for our model can be presented as follows:

$$\begin{aligned}
 & [\sum_{i=1}^2 A_i \lambda_i + B \mu_0 + \sum_{i=1}^2 C_i \mu_i] \pi_{m,n_1,n_2} = \\
 & \sum_{i=1}^2 D_i \mu_i \pi_{m,n_1-(1-\delta)Q_i,n_2-\delta Q_2} \\
 & + E \mu_0 \pi_{m-Q_0,n_1,n_2} \\
 & + \sum_{i=1}^2 F_i \lambda_i \pi_{m,n_1+(1-\delta),n_2+\delta} \\
 & + \sum_{i=1}^2 G_i \lambda_i \pi_{m+Q_i,n_1+(1-\delta),n_2+\delta}
 \end{aligned} \tag{2}$$

Where  $\delta$  is an operator that distinguishes two retailers. It takes 1 and 0 for the first and the second retailers respectively.

$$A_i = \begin{cases} 0 & (m = -\sum_{i=1}^2 Q_i \ \& \ n_{2-\delta} \geq R_{2-\delta} + 1) \\ & (m = -Q_i \ \& \ n_i \geq R_i + 1 \ \& \ n_{2-\delta} \geq R_{2-\delta} + 1) \\ & (m = -Q_i \ \& \ n_i \leq R_i \ \& \ n_{2-\delta} \leq R_{2-\delta}) \\ & (n_{2-\delta} = 0) \\ 1 & \textit{otherwise} \end{cases} \tag{3}$$

$$B = \begin{cases} 1 & (m \leq R_0) \\ 0 & \textit{otherwise} \end{cases} \tag{4}$$

$$C_i = \begin{cases} 0 & (m = -\sum_{i=1}^2 Q_i) \textit{ or } (m = -Q_i) \\ & (n_i \geq R_i + 1) \\ 1 & \textit{otherwise} \end{cases} \tag{5}$$

$$D_i = \begin{cases} 1 & (n_i \geq R_i) \ \& \ (n_i - Q_i \geq 0) \ \& \ (m \geq 0) \\ 0 & \textit{otherwise} \end{cases} \tag{6}$$

$$E = \begin{cases} 1 & (m - Q_0) \geq (-\sum_{i=1}^2 Q_i) \\ 0 & \textit{otherwise} \end{cases} \tag{7}$$

$$F_i = \begin{cases} 0 & (n_i = R_i + Q_i) \text{ or } (n_i = R_i) \\ & (n_{2-\delta} \geq R_{2-\delta} + 1 \ \& \ m = -\sum_{i=1}^2 Q_i) \\ & (n_{2-\delta} \geq R_{2-\delta} + 1 \ \& \ n_{2-\delta} \geq R_{2-\delta} + 1 \ \& \ m = -Q_i) \\ & (n_i \leq R_i \ \& \ n_{2-\delta} \leq R_{2-\delta} \ \& \ m = -Q_i) \\ & (n_i \leq R_i \ \& \ n_{2-\delta} \geq R_{2-\delta} + 1 \ \& \ m = -\sum_{i=1}^2 Q_i) \\ 1 & \text{otherwise} \end{cases} \tag{8}$$

$$G_i = \begin{cases} 1 & (n_i = R_i \ \& \ n_{2-\delta} \geq R_{2-\delta} + 1 \ \& \ m \geq -Q_i) \\ & (n_i = R_i \ \& \ n_{2-\delta} \leq R_{2-\delta} + 1 \ \& \ m \geq 0) \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

The left-hand side of equation (2) shows the output rate from the state  $(m, n_1, n_2)$ . Inside the bracket, the first term is the transition rate at which retailers receive the customer demand; the middle term is the transition rate at which replenishment orders are arrived at the warehouse; the last term is the transition rate at which replenishment orders are arrived at the retailers.  $A_i = 0$  where  $(i=1, 2)$ , defines the states in which satisfying the customer demand is impossible.  $B=1$ , defines the states in which the replenishment orders of the warehouse is possible.  $C_i = 0$  where  $(i=1, 2)$ , defines the states in which the replenishment orders of the retailers are impossible.

The right-hand side of equation (2) shows the input rate into the state  $(m, n_1, n_2)$ , where the first two terms present the transition rates caused by receiving the replenishment orders from the retailers and the warehouse, respectively. the last two terms present the transition rates due to satisfying demands.  $D_i = 1$  where  $(i=1, 2)$ , indicates receiving states of replenishment orders occur in retailers.  $E = 1$ , indicates receiving states of replenishment orders occur in warehouse.  $F_i = 0$  where  $(i=1, 2)$ , defines the states in which satisfying the customer demand is impossible.  $G_i = 1$  where  $(i=1, 2)$ , indicates the states in which satisfying the customer demand causes a retailer to place an order.

We can find the steady-state probabilities by solving the corresponding balance equations given in equation (2) and the normalizing constraint:

$$\sum_{m=-(Q_1+Q_2)}^{R_0+Q_0} \sum_{n_1=0}^{R_1+Q_1} \sum_{n_2=0}^{R_2+Q_2} = 1 \tag{10}$$

The steady-state probabilities are determined and will be used in performance measurement of the two-echelon inventory system.

**EVALUATING THE PERFORMANCE OF THE SYSTEM**

In this section, we present a cost structure considering three different operational cost factors, the long-run average inventory holding cost, the long-run average lost sales cost and the long-run average ordering cost. We will specify these cost factors in terms of the steady-state probabilities.

**Long-run average inventory holding cost**

Given the steady-state probabilities, the long-run average inventories in the warehouse and the retailers can be modeled, respectively as

$$E(I_0) = \sum_{m=0}^{R_0+Q_0} \sum_{n_1=0}^{R_1+Q_1} \sum_{n_2=0}^{R_2+Q_2} m \pi_{m,n_1,n_2} \quad (11)$$

and

$$E(I_i) = \sum_{m=-(Q_1+Q_2)}^{R_0+Q_0} \sum_{n_1=0}^{R_1+Q_1} \sum_{n_2=0}^{R_2+Q_2} n_i \pi_{m,n_1,n_2} \quad (12)$$

Then, the long-run average inventory holding cost,  $C_H$ , is determined by:

$$C_H = \sum_{i=0}^2 h_i \cdot E(I_i) \quad (13)$$

The first and second portion of  $C_H$  specifies the inventory holding cost of the warehouse and inventory holding cost of retailers, respectively.

### Long-run average lost sales cost

Stock out in retailers are lost sales. We use the steady-state probabilities obtained in the previous section to determine the long-run average lost sales for both retailers:

$$E(LS_1) = \lambda_1 \cdot p(n_1 = 0) = \lambda_1 \cdot \sum_{m=-(Q_1+Q_2)}^{R_0+Q_0} \sum_{n_2=0}^{R_2+Q_2} \pi_{m,0,n_2} \quad (14)$$

and

$$E(LS_2) = \lambda_2 \cdot p(n_2 = 0) = \lambda_2 \cdot \sum_{m=-(Q_1+Q_2)}^{R_0+Q_0} \sum_{n_1=0}^{R_1+Q_1} \pi_{m,n_1,0} \quad (15)$$

Then we can specify the total long-run average lost sales cost as:

$$C_{LS} = \sum_{i=1}^2 l_i \cdot E(LS_i) \quad (16)$$

### Long-run average ordering cost

The average number of replenishment orders (ordering) per unit time at both retailers is equal to (Saffari et al., 2011):

$$E(RP_i) = \frac{\bar{\lambda}_i}{Q_i} \quad (17)$$

in which  $\bar{\lambda}_i$  is the effective arrival rate that can be calculated through  $\bar{\lambda}_i = \lambda_i (1 - p(n_i = 0))$ .

The average number of replenishment orders per unit time at the warehouse is equal to:

$$E(RP_0) = \frac{\sum_{i=1}^2 \lambda_i \cdot p(n_i = R_i + 1)}{Q_0} \quad (18)$$

If on hand inventory at retailer  $i$  is  $R_{i+1}$ , then an entry of a customer reduces the on-hand inventory at the retailer to  $R_i$ , and the retailer  $i$  places an order to the warehouse. Thus, the average number of the orders that retailer  $i$  sends to the warehouse (average demand from warehouse) will be  $\lambda_i \cdot p(n_i = R_i + 1) / Q_i$ . So, the long-run average ordering cost,  $C_{RP}$ , is determined by:

$$C_{RP} = \sum_{i=0}^2 r_i \cdot E(RP_i) \quad (19)$$

**Total cost of system**

We will use the sum of the inventory holding cost and the lost sales cost and ordering cost to evaluate the performance of the two-echelon inventory system. Therefore, the total cost is defined as  $C_H + C_{LS} + C_{RP}$ . The decision variables are the batch size and reorder point at each level. Therefore, we can view total long-run cost as a function of  $R_i, Q_i, (i = 0, 1, 2)$ , that is,

$$\begin{aligned}
 TC = & h_0 \cdot \sum_{m=0}^{R_0+Q_0} \sum_{n_1=0}^{R_1+Q_1} \sum_{n_2=0}^{R_2+Q_2} m \pi_{m,n_1,n_2} \\
 & + \sum_{i=1}^2 h_i \cdot \sum_{m=-(Q_1+Q_2)}^{R_0+Q_0} \sum_{n_1=0}^{R_1+Q_1} \sum_{n_2=0}^{R_2+Q_2} n_i \pi_{m,n_1,n_2} \\
 & + l_1 \cdot \lambda_1 \cdot \sum_{m=-(Q_1+Q_2)}^{R_0+Q_0} \sum_{n_2=0}^{R_2+Q_2} \pi_{m,0,n_2} \\
 & + l_2 \cdot \lambda_2 \cdot \sum_{m=-(Q_1+Q_2)}^{R_0+Q_0} \sum_{n_1=0}^{R_1+Q_1} \pi_{m,n_1,0} \\
 & + r_0 \cdot \frac{\sum_{i=1}^2 \lambda_i \cdot p(n_i = 0) \cdot p(n_i = R_i + 1)}{Q_0} \\
 & + \sum_{i=1}^2 r_i \cdot \frac{\lambda_i \cdot (1 - p(n_i = 0))}{Q_i}
 \end{aligned}$$

*Subjecto :*

$$\begin{aligned}
 Q_i & \geq R_i \quad Q_0 \geq k Q_i \quad R_0 \geq L Q_i \\
 k, L & \in N \quad Q_i, R_i \geq 0 \quad i = 1, 2
 \end{aligned}$$

(20)

The objective in this paper is to find the decision variables that minimize this total cost.

**SOLUTION PROCEDURE**

Our proposed model in the previous section is a nonlinear integer programming (NIP) problem; considering the nonlinearity and the aim of our model that tries to search the space for the best solutions, we are lead to meta-heuristic algorithm. GA has been successfully applied to some inventory control problems, however, the application of GA to multi-echelon inventory systems are still rare[11]. In this paper we decided to use GA.

**The proposed GA method**

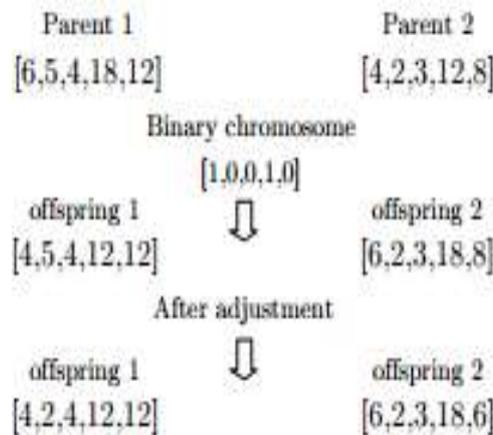
One of the most important factors for successful implementation of GA is designing a more suitable chromosomal structure. In this paper, the chromosomal solution consists of a matrix-vector with one row and five columns. The first element is to indicate the batch size of Retailers. Note that the batch size is assumed to be identical for both retailers ( $Q_r = Q_1 = Q_2$ ). The second and third elements represent reorder points of both retailers. The fourth and the fifth elements, respectively, represent the batch-size and reorder point at the warehouse. Chromosomal structure is depicted in Fig- 2 .

$$[Q_r, R_1, R_2, Q_0, R_0]$$

**Fig- 2: Chromosomal structure**

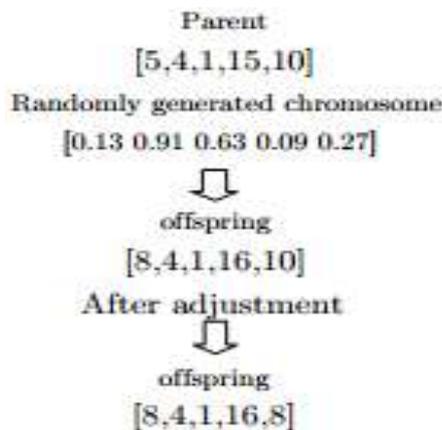
First of all, the initial population of solutions is generated randomly.in the next step a fitness function is provided to evaluate the chromosomes of each generation. Parents are selected using one of the most popular selection methods called roulette-wheel. In this method, the parents are chosen based on the probability distribution of their fitness

value. Although better individuals will have a higher selection probability, all individuals in the population will have a chance to be selected, and then Reproduction is carried out by using crossover and mutation operators on the selected parents to produce new offspring. As an example, the crossover operation is performed as given in Fig-3.



**Fig-3: An illustration of the crossover operation**

To explore new solutions, the mutation operator performs random alterations in chromosome genes by a predetermined mutation rate of  $P_m$ . For the mutation operation of this paper, first a random chromosome whose components are between 0 and 1 is created and applied to the selected parent. Then, the parent genes that correspond to values less than pm are mutated within the boundaries of their corresponding variable. Fig-4 illustrates a mutation operation in which pm is set at 0.2.



**Fig-4: An illustration of the mutation operation**

**Parameters**

Parameters in GA consist of a probability of performing crossover called crossover rate indicated by  $P_c$  and a probability of performing mutation called mutation rate indicated by  $P_m$ . The number of chromosomes called population size, plays the main role in the run time of the algorithm to reach the near-optimal solution. These parameters have a crucial role in the performance of GAs.

**NUMERICAL ILLUSTRATION**

In this section, we present a numerical example by considering different values of parameters to show how the model is working. The basic parameter values are shown in

Table 1. The capacities of production transportation for each echelon are 15 and 30 for retailers and warehouse, respectively. To demonstrate the performance of the proposed genetic algorithm, we solved an example with two distinct methods of the direct search and proposed genetic algorithm. Minimum cost and optimal ordering policy obtained by both methods are shown in Table 2. To solve the model for different values of the batch size and the reorder point, we used the MATLAB programming language.

**Table 1: Base parameter values**

Row	$\lambda_1$	$\lambda_2$	$\mu_0$	$\mu_1$	$\mu_2$	$h_0$	$h_1$	$h_2$	$l_1$	$l_2$	$r_0$	$r_1$	$r_2$
1	2	9	4	7	11	4	3	2	25	15	20	5	3
2	5	13	4	7	11	4	3	2	25	15	20	5	3
3	5	9	1	7	11	4	3	2	25	15	20	5	3
4	5	9	4	10	11	4	3	2	25	15	20	5	3
5	5	9	4	7	7	4	3	2	25	15	20	5	3
6	5	9	4	7	11	1	3	2	25	15	20	5	3
7	5	9	4	7	11	4	5	2	25	15	20	5	3
8	5	9	4	7	11	4	3	1	25	15	20	5	3
9	5	9	4	7	11	4	3	2	75	15	20	5	3
10	5	9	4	7	11	4	3	2	25	75	20	5	3
11	5	9	4	7	11	4	3	2	25	15	150	5	3
12	5	9	4	7	11	4	3	2	25	15	20	10	3
13	5	9	4	7	11	4	3	2	25	15	20	5	24

**Table 2: Performance GA in compared with direct search**

Row	DS						TC	GA						TC
	$R_0$	$Q_0$	$R_1$	$Q_1$	$R_2$	$Q_2$		$R_0$	$Q_0$	$R_1$	$Q_1$	$R_2$	$Q_2$	
1	6	6	1	6	1	6	57.15	5	5	1	5	1	5	57.19
2	7	7	2	7	2	7	73.85	7	7	2	7	2	7	73.85
3	18	9	1	9	1	9	93.18	18	9	1	9	1	9	93.18
4	8	8	1	8	1	8	64.03	7	7	2	7	2	7	64.61
5	7	7	2	7	2	7	67.21	7	7	2	7	2	7	67.21
6	7	7	2	7	2	7	48.64	8	8	1	8	1	8	49.28
7	6	6	1	6	1	6	73.42	6	6	1	6	1	6	73.42
8	7	7	2	7	2	7	58.77	6	6	2	6	2	6	61.00
9	7	7	2	7	2	7	68.73	7	7	2	7	2	7	68.73
10	8	8	3	8	3	8	78.62	8	8	3	8	3	8	78.62
11	0	0	1	0	1	10	89.43	9	9	2	9	2	9	92.92
12	7	7	2	7	2	7	67.44	8	8	1	8	1	8	67.55
13	8	8	1	8	1	8	77.05	7	7	2	7	2	7	78.52

$Q_r = [1-15], Q_0 = [1-30],$  population size= 50,  $p_c = 0.8, p_m = 0.15,$  Number of iterations= 40

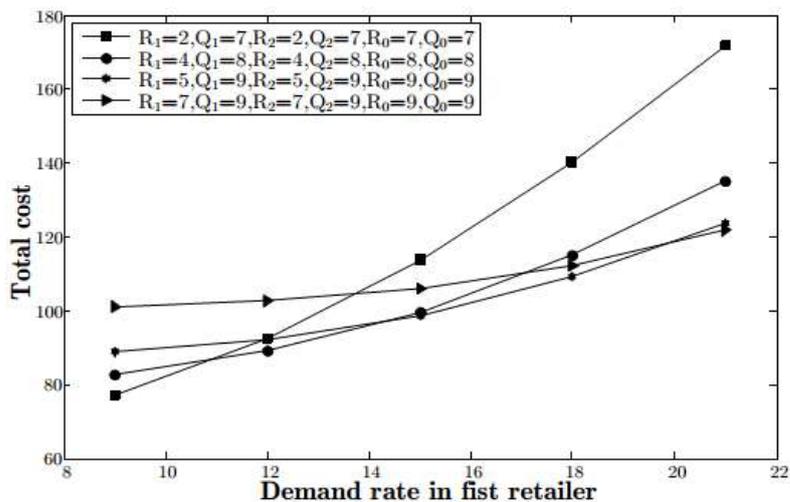


Fig- 5: The changes of the total cost of types of ordering policy with respect to the demand rate

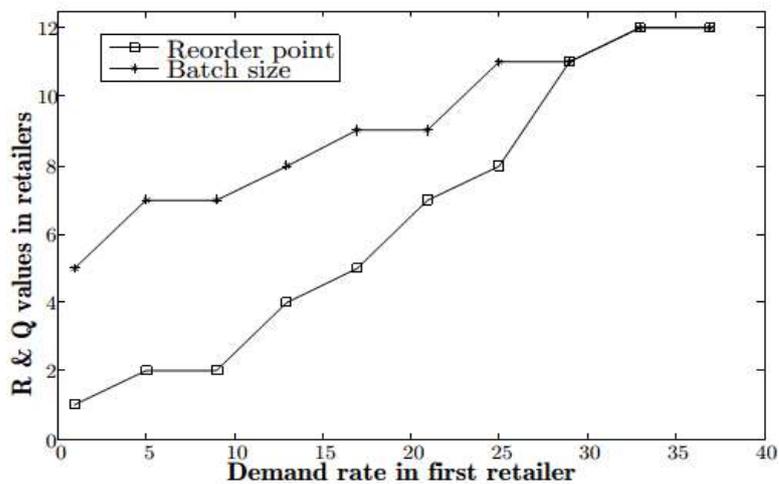


Fig-6: The changes of the reorder point and batch size values in retailers with respect to the demand rate

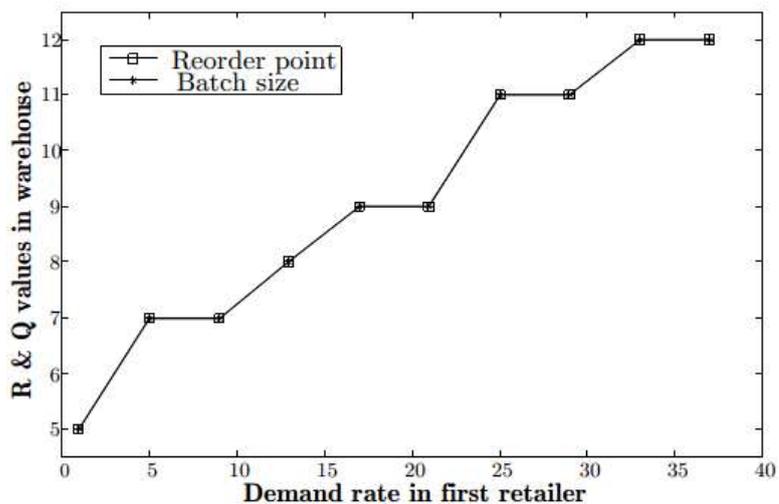


Fig-7: The changes of the reorder point and batch size values in warehouse with respect to the demand rate

For more explanation of the result of the problem we used three charts.

shows the changes of the total cost caused by changing the ordering policy and the demand rate in a retailer. Fig-6 and Fig-7 illustrate the effect of the changes of the demand rate on the reordering policy for the retailers and the warehouse, respectively.

### CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

In this research, a new model for a two-echelon inventory system based on inventory policy of  $(R, Q)$  was developed to determine the optimal values of  $(R, Q)$  in both echelons, so that the total cost is minimized. Markov chain and genetic algorithm were applied to solve the model. Finally a numerical example was used to illustrate the performance of the model while changing the system parameters and also the performance of genetic algorithm in this model was presented. Future research can extend the model by considering different assumptions such as allowing lateral transfer between retailers, considering more than two retailers, allowing direct submission from the external supplier to retailers while warehouse faces stock out, considering multi-commodity and budget limitations.

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