

Risks Measurement and Analysis of Shanghai Stock Market Index Based on GARCH-VaR Model

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Abstract: This paper adopts conditional heteroscedasticity of GARCH and variance-covariance of VaR calculation method to measure VaR. This paper builds through rate of monthly return in Shanghai stock market within ten years. EGARCH-GED Model is selected to calculate VaR value at risks of Shanghai stock market under three confidence levels such as 90%, 95% and 99%.

Keywords: VaR , GARCH Model, Shanghai Composite Index

INTRODUCTION

In recent years, global financial market develops rapidly, at the same time, investors have been conscious of uncertainty of financial crises. VaR method is widely used by institutions such as financial institution due to its simplicity and practicality. China Stock Market started relatively late. Nowadays, it's in development phrase, and stock returns take on volatility clustering. GARCH Model not only can reflect correlation of series of stock returns, but also reflects characteristics[1] of time-varying of market. Therefore, GARCH Model is adopted for calculating VaR value to assess its value at risk.

Calculation of VaR and GARCH Model

VaR refers to an asset or combination of assets' maximum possible losses[2] within a certain period in the future under a confidence level of certain probability. As for calculation of VaR under normal distribution, it may set V_0 as the amount of certain investment portfolio. If investment return of entire investment period is r , till the end of investment period, value of investment portfolio is $V = V_0(1+r)$. Expected rate of return of investment period is μ and volatility of rate of return is σ . The minimum value of investment portfolio under confidence level α is $V^* = V_0(1+r_a)$, and the minimum rate of return of investment under confidence level α is r_a .

$$VaR_r = E(V) - V^* = -V_0(r_a - \mu) \quad (1)$$

When rate of return is under normal distribution $r_t \sim N(\mu, \sigma^2 \Delta t)$, therefore, $-Z_\alpha = \frac{r_a - \mu}{\sigma \sqrt{\Delta t}}$ we can get

$$VaR_r = V_0 Z_\alpha \sigma \sqrt{\Delta t} \quad (2)$$

Supposing that mean equation of $\{r_t\}$ is represented with ARMA(m,n) as: $y_t = c + \sum_{i=1}^m \phi_i y_{t-i} + \sum_{j=1}^n \theta_j u_{t-j} + u_t$, of which u_t is perturbation element of period of t, submitting mean value is 0, variance is σ_t^2 , and degree of freedom is distribution of V .

In 1986, Bollerslev put forward GARCH Model[3] based on ARCH Model, which was the important development of ARCH Model, and largely reduced complexity of models.

The equation of GARCH is:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

Volatility on rate of return is asymmetric, in order to decrease its asymmetric impact, in 1991, Nelson put forward EGARCH Model [4] which was also called Index GARCH Model.

$$\text{The equation of EGARCH is } \ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^p \alpha_i * \left| \frac{u_{t-i}}{\sigma_{t-j}} \right| + \sum_{j=1}^q r_j * \ln(\sigma_{t-j}^2) + \sum_{k=1}^m c_k \left| \frac{u_{t-k}}{\sigma_{t-k}} \right|$$

TTGARCH Model is also called Threshold ARCH Model, which was initially put forward by J.M. Zakoian in 1990. Threshold of TGARCH Model was set through dummy variable, so as to distinguish different impacts[5] of positive and negative shocks on conditional volatility.

$$\text{The equation of TARCH Model is: } \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i u_{t-i}^2 + \sum_{j=1}^q r_j \sigma_{t-j}^2 + \sum_{k=1}^m c_k d_{t-k} u_{t-k}^2$$

$$d_{t-k} = \begin{cases} 1, & u_{t-k} < 0 \\ 0, & u_{t-k} \geq 0 \end{cases}$$

Meeting the following conditions

Parameter m represents number of thresholds, and d_{t-k} represents dummy variable. Impacts on conditional variance of positive shock $u_{t-k} \geq 0$ and negative shock $u_{t-k} < 0$ are different. Considering the condition of $p = q = 1$, at that time, positive shock is a shock with α_1 times while negative shock is a shock with $(\alpha_1 + c_1)$. If $c_1 > 0$, it shows that there exists leverage effect and it increases amplitude of fluctuation. On the contrary, if $c_1 < 0$, it shows that the main result of asymmetric effect is to decrease the amplitude of fluctuation.

CALCULATION OF GARCH-VAR MODEL

This dissertation adopts variance-covariance method to calculate VaR value. VaR value is calculated with conditional variance $\sqrt{\sigma_t}$ of GARCH Model. $VaR = P_{t-1} Z_\alpha \sqrt{\sigma_t} (3)$, P_{t-1} is stock closing price in $t-1$ period.

Test of Calculation Effect

As for test of VaR value's calculation effect, we adopt Failure Test Method put forward by Kipuec. Construction statistic is $LR = 2 \ln \left[(1-P)^{T-N} * P^N \right] - 2 \ln \left[(1-P^*)^{T-N} * P^{*N} \right]$, of which $P^* = 1 - \alpha$ is expected probability of

each failure, $P = \frac{T}{N}$ is actual failure frequency, T is actual days of failure, and N is actual days of investigation. Supposing

$P = P^*$, $LR \sim \chi^2(1)$ shall be distributed as assumed condition.

EMPIRICAL RESEARCH

Select monthly closing price of Shanghai composite Stock from Jan. 2004 to Sep. 2014 as research object, and adopt the form of rate of return of natural logarithm as rate of monthly return, monthly closing price of period of t is P_t , and process rate of logarithmic return $r_t = \ln p_t - \ln p_{t-1}$ by Eviews software.

Statistical Analysis of Characteristic

Statistical analysis of yield shows that: Average value of return series of Shanghai Composite Index is 0.003095, skewness is -0.583238 and kurtosis is 4.366674, also, *Jarque - Bera* value is 17.21848 > 0.05, and distribution

probability corresponding to χ^2 distribution approaches 0, so the assumption of normal distribution is rejected. To put it in another way, the return series of Prev doesn't subject to assumption of normal distribution, displaying a asymmetry and high-peak & heavy tails in feature.

Assess stability of earning ratio to perform ADF test. And test result indicates that the series is stable under significant differences in level 1%, 5% and 10%, and there is no unit root, which suggests that return rate of Shanghai stock is under a state of stability.

Look through the data and confirm whether there are relationships between residuals series of yield, at the same time, the self-phased property shall also be tested. Use LB-Q Method to perform correlation test for Auto-Correlation Function (AC) and Partial Correlation Function (PAC).

From graphs of function AC and PAC in 15-order lag, we can find the coefficients of correlation are significant. While in 2-order and 4-order, Q statistic test of return rate indicates that there is a correlation between series. According to Jenkins - Box modeling theory, we know that to choose ARMA (m, n) model is more appropriate. Further to determine lag order m and n, we build a fitting model in which the value of m and n fall in 0 to 4, in the meanwhile, we compare residual, AIC and SC by model fitting. Finally, we get below mean equation:

$$r_t = -0.387r_{t-2} - 0.4106r_{t-4} + 0.6147u_{t-2} + 0.8728u_{t-4} + u_t$$

In residual value fitting, AIC and SC reach to minimum value basically. To perform Q statistic test for the model, we find there is no correlation between residual series. In order to verify conditional heteroscedasticity in volatility, we perform an ARCH-LM test for return rate in 7-order, then we find the test result rejects original assumption under confidence level in 95%, which means Shanghai Composite Index has a property of conditional heteroscedasticity.

Respectively, we assume residual series is subject to t and GED distribution, then we find that AIC and SC of EGARCH (1,1) distribution under GED distribution matches with maximum likelihood function L . Perform ARCH-LM test on square sequence of residuals of the model, and result suggests that there is no conditional heteroscedasticity for logarithm yield sequence. Result of model fitting under EGARCH-GED (1,1) is as follows:

Table-1: Result of model fitting under EGARCH-GED

Model	W	α	β	γ
EGARCH(1,1)	0.0567	-0.1313	0.9937	0.1866
Probability (P)	0.0000	0.0000	0.0000	0.0000

From this table, we can find all parameters are significant under 1%, 5% and 10%. $\alpha + \beta < 1$ indicates that the return series has the property of persistence volatility.

TEST AND CALCULATION OF VARs.

According to reference estimated value and EViews method, we can work out conditional variance, further we get standard deviation. Also, based on equation (2) we work out VaRs. Here we have respectively worked out VaRs in three confidence levels, they are 90%, 95% and 99%.

Table-2: VAR Value

Confidence Level	Minimum VaR	Maximum VaR	Average VaR	Failures Count	Standard s Count	Failure Rate	Failure rate of standards Count	LR Value
90%	61.76	1224.03	246.6	7	124	5.6%	10%	3.05
95%	75.75	1501.36	302.48	5	124	4%	5%	0.26
99%	99.2	1996.01	396.1	3	124	2.4%	1%	1.8

From above table, we can see that there is a significant difference in VaRs under three confidence levels. From the point of Failure Rate and Failures, we can see VaRs calculated under EGARCH-GED model approaches to standard level under confidence level in 95%. While for confidence level in 99%, number of failures of VaR under the model exceeds

standard level. Test in Failed Test Method, we can see LR value is greater than $\chi_{0.1}^2(1) = 2.07$ under confidence level in 90%, so the original assumption is rejected. Under confidence level in 95% and 99%, LR values are less than $\chi_{0.05}^2(1) = 3.48$ and $\chi_{0.01}^2(1) = 6.63$, so the original assumption is accepted. Further, it suggests that we can get reliable VaRs by calculation from EGARCH(1,1) model under confidence level in 95% and 99%. Therefore, the risk model EGARCH(1,1) under GED distribution can better curve features of monthly stock yield.

CONCLUSIONS

(1) From certain data above, monthly closing price of Shanghai Composite Index displays high-peak and heavy-tail as well as volatility in feature. Through comparison and analysis of AIC, SC, fitting value R^2 as well as log likelihood L, choose model GARCH under GED distribution, finally by model solution we get VaR of Shanghai stock under confidence level in 90%, 95% and 99% .

(2) From calculated VaR, it shows that the higher confidence level is, the greater calculated VaR value of Shanghai is. This is because a small probability makes it difficult to represent accuracy, so does it to estimation.

(3) From VaR value calculated from EGARCH(1,1) model under GED distribution, confidence level in 95% and 99% passed the test by Failed Test Method, and calculated risk value VaR of Shanghai is reliable.

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