

## Based on Copulas Connect Theory Analysis of the Risk of Infection

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**Abstract:** In this paper to undertake Preliminary research on correlation of Gold, Crude Oil and the Dow Jones Industrial Average, have the same data source. Copulas connect model is established to solve the related structures between the financial time series. Can directly find correlation between the sequences. So can be used as a research and analysis prediction method to system comprehensive analysis of Crude Oil as the investment value of investment products, and has a guiding significance for other related analysis.

**Keywords:** Copula, Risk Analysis, Non-parametric Estimation, Correlation.

### INTRODUCTION

The US subprime mortgage crisis caused by the Wall Street storm, finally evolved into a global financial crisis. In 1873, the German and Austrian foreign credit suspended suddenly, in the United States jay cooke companies operating difficulties; In 1890, London Bahrain Brothers Investment Bank arouse Argentina payment crisis, New York haven financial crisis in October of the same year, national and regional economic crisis continued until 1893; Along with the expansion of the international hot money in the 1990s, the international monetary and financial crisis broke out frequently. In June 1997, Thailand float sparked a throughout the southeast Asian financial turmoil. 2007-2008 global financial crisis broke out again, and led to many large financial institutions collapse or by the government to take over. In 2014, the stock market changes again, crude oil prices continue to reduce, which cause the extensive concern of investors.

Due to the nature of gold, gold as a risk can be measured the relative standards, used to measure the risk of crude oil. In this paper to undertake Preliminary research on correlation of Gold, Crude Oil and the Dow Jones Industrial Average, have the same data source.

### THEORETICAL ANALYSIS

Copulas connect theory was born out of research to determine the edge distribution function of the process, in 1990 to 2004, a series of edge correlation and solution of the distribution function of modeling study, copulas connect to induction and development theory. Studies of copulas connect theory first appeared in the 1950s, Jeo 1 and Nelson 2 and so on, the system introduced the relevant theorems and properties of copulas connect theory development initial period, Hanming 3, Claudia 4 and Scholzel 5 respectively introduce copulas connect theory gradually mature process in detail. In earlier research, Yan Hua Wei 6, Junpeng Luo 6, Dewen Yi 8, etc. ,in their PhD thesis to conduct a comprehensive induction for it.

Theory of copulas connect research have good results, before copulas connect theory is introduced into China, copulas connect in China mainly concentrated in the development of applications, theoretical innovation and limitations in the parameter to estimate the accurate degree of fitting method, and the model test on the improvement. Copulas connect theory research mainly has the earliest do Yan Hua Wei, Shiyong Zhang 9 and Daoji Shi, Yinchu Yao 10, the theory of research are most on the basis of the application. Copulas connect theory at the beginning of the development of domestic, YanHua Wei Shiyong Zhang 1113 in the 2006 years ago, the main use of copulas connect theory combined with the improved method, through the establishment of copulas connect model to analyze the financial market structure.

**Theorem 2.1** (Sklar theorem). Establish joint distribution function  $H$  has the edge distribution function  $F$  and  $G$ . So, there is a Copula  $C$ , for any  $x, y \in \bar{R}$ , there are:

$$H(x, y) = C(F(x), G(y)) \quad (1)$$

If  $F$  and  $G$  are straight,  $C$  is the only; Otherwise, the uniqueness of  $C$  on the set. On the other hand, if  $C$  is copulas connect,  $F$  and  $G$  is the distribution function, so there are define the function of  $H$ , is a joint distribution function and its edge distribution function  $F$  and  $G$ .

Elliptical copulas connect function is a class of elliptic contour distribution function, the most common elliptical copulas connect function is Gaussian (Gaussian) copulas connect function and t-copulas connect function. Elliptical copulas connect function has the advantage that can construct different distributions of the edge of the dependent degree of copulas connect function, and the disadvantage is that the distribution function is not closed thought forms, and are the radial symmetry. t- Copulas connect function (2-dimensional) is:

$$C(u, v; \rho, \kappa) = \int_{-\infty}^{T_{\kappa}^{-1}(u)} \int_{-\infty}^{T_{\kappa}^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(1 + \frac{s^2 - 2\rho st + t^2}{\kappa(1-\rho^2)}\right)^{\frac{-(\kappa+2)}{2}} ds dt \tag{2}$$

Archimedean copulas connect function has many advantages, can be found from expression Archimedean copulas connect function with symmetry. Archimedean copulas connect function can be combined, for example:  $C(u_1, C(u_2, u_3)) = C(C(u_1, u_2)u_3)$ . Archimedean copulas connect function is easy to calculate, so in more empirical analysis usually choose Archimedes copulas connect function to establish model.

Binary Archimedean copulas connect function is defined as:

(1) Gumbel Copula

$$C(u, v) = \text{MAX}((u^{-\theta} + v^{-\theta} - 1)^{\frac{-1}{\theta}}, 0) \tag{3}$$

$\theta$  is  $[-1, \infty) \setminus 0$ , generator is  $\varphi(\omega) = (-\log(\omega))^{-\theta}$ , Kendall's Tau is Similarity index, estimated value is  $1 - \frac{1}{\theta}$ .

(2) Clayton Copula

$$C(u, v) = \varphi^{[-1]}(\varphi(u) + \varphi(v)) \tag{4}$$

$\theta$  is  $[1, \infty)$ , generator is  $\varphi(\omega) = \theta^{-1}(\omega^{-1} - 1)$ , Kendall's Tau is Similarity index, estimated value is  $\frac{\theta}{(\theta + 2)}$ .

(3) Frank Copula

$$C(u, v) = -\theta^{-1} \log\left(1 - \frac{(1 - e^{-\theta u})(1 - e^{-\theta v})}{(1 - e^{-\theta})}\right) \tag{5}$$

$\theta$  is  $\theta \neq 0$ , generator is  $\varphi(\omega) = -\log\left(\frac{e^{-\theta\omega} - 1}{e^{-\theta} - 1}\right)$ , Kendall's Tau is Similarity index, estimated value is  $1 + \frac{4\{D_1(\theta) - 1\}}{\theta}$ .  $D_1$  is  $D_1(\theta) = \frac{1}{\theta} \int_0^{\theta} \frac{t}{\exp(t) - 1} dt$ .

**CONSTRUCTION OF MODEL**

Copulas connect function can independently analyze the related structure between the marginal distribution and the reflect of random variable. Therefore, construction of copulas connect function is normally need two steps.

Step 1: Build edge distribution function to fitting a single change characteristics and distribution characteristics of the base variable;

Step 2: Choose a copulas connect function which can better performance dependent structure between random variables.

Copulas connect function was established for the building of model, parameter estimation and nonparametric estimation can be divided into two ways. Parameter estimation is to solve the parameters in the model, the parameter estimation is the value of copulas connect function is estimated based on the observed value. Nonparametric estimation

does not require any assumption and parameter estimation, estimation results depends entirely on observations of randomness, and existing value is worth repeating in the future. Due to the large amount of data, this paper chooses the parameter estimation as analysis method. After the copulas connect model was set up, need to test the model.

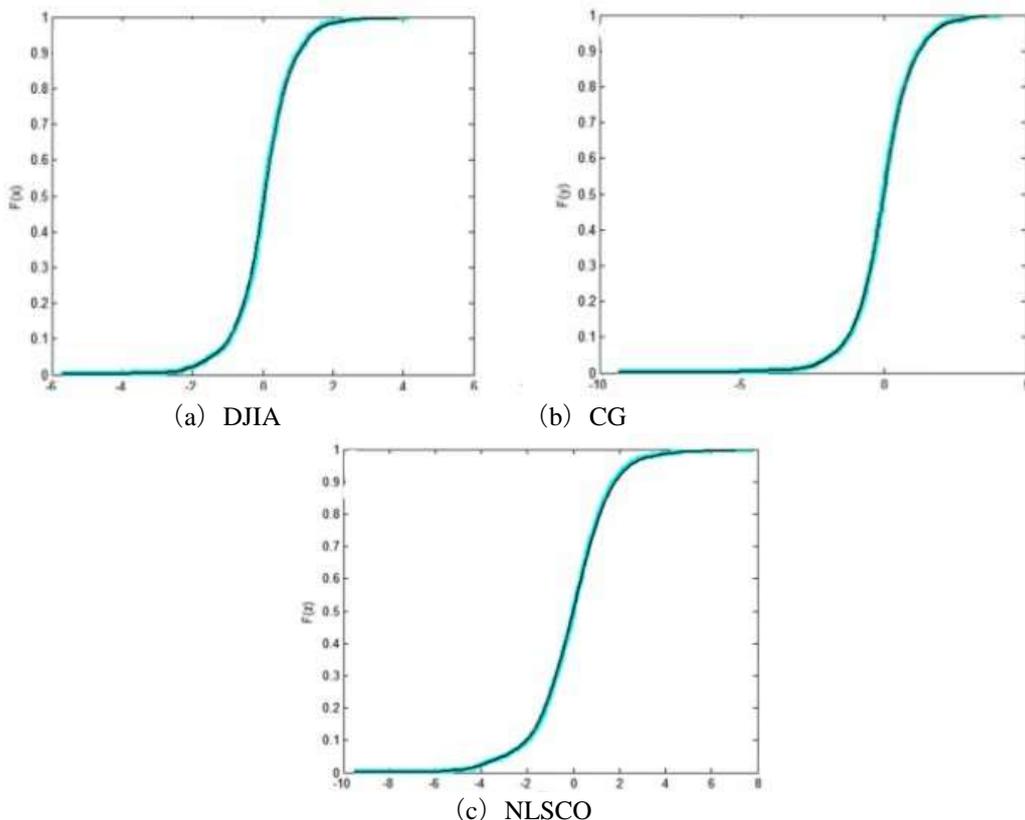
**EMPIRICAL ANALYSIS**

Using GARCH-t (1, 1) and GARCH-GED (1, 1) model to describe each marginal distribution, choosing the model as the marginal distribution model of the sample when the model has a high value of likelihood. Table 1 shows the sample selection of marginal distribution model and its parameter estimation and test results.

**Table-2: Parameter estimates and marginal distribution model test results**

Observation Sequence	$\mu$	$\omega$	$\alpha$	$\beta$	$k$	Likelihood
DLCG	-0.03629 (1.1202)	0.005537 (0.0047)	0.027845 (0.0084)	0.967719 (0.0095)	4.0130 (0.4969)	-1481.849
DLDJIA	0.03372 (0.8868)	0.034534 (0.0113)	0.163694 (0.0346)	0.793465 (0.0384)	6.8981 (1.7792)	-1194.506
DLNLSCO	-0.06888 (1.6858)	0.019783 (0.0121)	0.060471 (0.0148)	0.934257 (0.0160)	7.7519 (1.9074)	375.6006

In the process of marginal distribution fitting, testing the vast majority of existing distribution model, in which GARCH-t (1, 1) model to estimate the marginal distribution fitting effect is better than the other. But the edge of the fitting of distribution coefficient of the sum of more than 1, and the standard deviation value is bigger, estimated parameters is not significant, it is used to describe the yield sequence on the edge of the distributed is still not very ideal. So, using nonparametric method fitting edge distribution function. Choose kernel distribution estimation method and visual representation by generating empirical distribution function and kernel distribution estimation comparison chart.



**Fig-1: Observation data yield frequency histogram estimation and nuclear distribution diagram**

Through the above you can see that the empirical distribution function and kernel distribution estimation figure almost overlap, can better express the information observed value. Estimated the parameters of the copulas connect model, based on generating new sequence.

The edge of the three sequence distribution, nuclear density estimation is used to estimate the parameter value. And the established copulas connect model corresponding to the relationship about three groups of Spearman rank correlation coefficient and Kendall rank correlation coefficient, compared with data of rank correlation coefficient after transformation. The U for DLDJIA and DLGC correlation analysis sequence, V for DLDJIA and DLNLSCO sequence of correlation analysis, W for DLGC and DLNLSCO sequence of correlation analysis. Test results are shown in table below.

**Table 2: Rank correlation coefficient of the five copulas connect model**

Rank Correlation Coefficient	Sequence	The Original Sequence	Gaussian	t	Gumbel	Clayton	Frank
Spearman	U	-0.0096	-0.0207	-0.0105	0.0089	0.01550.15	-0.0126
	V	0.1664	0.1794	0.1785	0.1524	23	0.1803
	W	0.1616	0.1473	0.1679	0.1623	0.1192	0.1806
Kendall	U	-0.0042	-0.0138	-0.0072	0.0059	0.0103	-0.0084
	V	0.1134	0.1201	0.1205	0.1024	0.1019	0.1207
	W	0.1140	0.0985	0.1148	0.1091	0.0796	0.1209

**Table 3: Square Euclidean distance of five kinds of copulas connect model**

Rank Correlation Coefficient	Gaussian	t	Gumbel	Clayton	Frank
U	103.5004 *	104.4165	104.7845	104.8403	103.8270
V	126.0051	126.1889	125.9851	123.3085 *	126.0478
W	126.4189	128.0088	128.4369	123.9664 *	127.9906

Through the above you can see, compared with five copulas connect estimates of the model and experience of copulas connect estimates. In correlation analysis DLDJIA and DLGC, Gaussian copulas connect function can better describe DLDJIA and DLGC related degree; Based on the analysis of correlation between DLDJIA and DLNLSCO structure, Clayton copulas connect function can better describe DLDJIA and DLNLSCO correlation degree; In DLGC and DLNLSCO correlation analysis, Clayton copulas connect function can better describe the relationship between the DLGC and DLNLSCO.

Due to the particularity of gold, gold can be assumed to be relative standard of risk measurement to measure the risk of crude oil. Analysis methods using t-copula connect function model, binary t-copula connect function of the linear correlation coefficient ( $\hat{\rho}$ ) and estimates of the degrees of freedom ( $\hat{\kappa}$ ) as follows:

$$\hat{\rho} = \begin{pmatrix} 1.0000 & 0.1793 \\ 0.1793 & 1.0000 \end{pmatrix}, \hat{\kappa} = 5.2316 \approx 5 \tag{6}$$

Take  $\hat{\rho}$ ,  $\hat{\kappa}$  the t-Copula connect function model

$$C(w, v; \rho, \kappa) = \int_{-\infty}^{T_5^{-1}(w)} \int_{-\infty}^{T_5^{-1}(v)} \frac{1}{2\pi\sqrt{1-0.1793^2}} \exp\left(1 + \frac{s^2 - 2 \times 0.1793st + t^2}{5(1-0.1793^2)}\right)^{-\frac{(5+2)}{2}} dsdt \tag{7}$$

Correlation between DLDJIA and DLNLSCO, analysis by t-copula connect function model, binary t-copula connect function of the linear correlation coefficient ( $\hat{\rho}_1$ ) and degrees of freedom ( $\hat{\kappa}_1$ ) estimate value as follows:

$$\hat{\rho}_1 = \begin{pmatrix} 1.0000 & 0.1882 \\ 0.1882 & 1.0000 \end{pmatrix}, \hat{\kappa}_1 = 12.2537 \approx 12$$

Take  $\hat{\rho}_1$ ,  $\hat{\kappa}_1$  to binary t-copula connect function model can be get:

$$C(u, w; \rho, \kappa) = \int_{-\infty}^{T_{12}^{-1}(u)} \int_{-\infty}^{T_{12}^{-1}(w)} \frac{1}{2\pi\sqrt{1-0.1882^2}} \exp\left(1 + \frac{s^2 - 2 \times 0.1882st + t^2}{12(1-0.1882^2)}\right)^{-\frac{(12+2)}{2}} dsdt \tag{8}$$

## CONCLUSION

This article carries on before for Gold, Crude Oil and The Dow Jones Industrial Average research base, combination of copulas connect model set up their correlation degree. Due to the nature of gold, gold as a risk can be measured the relative standards, used to measure the risk of crude oil. For the above findings, the Dow Jones Index can directly affect the price of Crude Oil, has guiding effect to the price of Crude Oil, analyze its relevance is advantageous to the analysis of Crude Oil price fluctuations. Due to the risk analysis and correlation analysis are reflected, through two copulas connect model can be more intuitive analysis whether the current Crude Oil production has high investment value. So can be used as a research and analysis prediction method to system comprehensive analysis of Crude Oil as the investment value of investment products, and has a guiding significance for other related analysis.

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