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Research Article

Classification of All Single Travelling Wave Atom Solutions to the (2 +1)-Dimensional Nonlinear KdV System

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Abstract: By the polynomial complete discrimination system, we give the classification of all single travelling wave atom solutions to the (2 +1)-dimensional nonlinear KdV system.
Keywords: travelling wave solution; complete discrimination system; the (2 +1)-dimensional nonlinear KdV system.

INTRODUCTION

Nonlinear partial differential equations play an important role in applied mathematics, physics and engineering. A lot of methods, such as the tanh method [1-2], the bifurcation theory and the method of phase portraits analysis [3], qualitative theory of polynomial differential system [4-5], Exp-Function method [6] and so on, have been proposed to solve these equations. Recently, Liu [7-9] introduced a simple and efficient method to give the classification of all single travelling wave atom solutions to some equations. If a nonlinear equation can be directly reduced to the integral form as follows:

$$\pm(\xi-\xi_0)=\int\frac{\mathrm{d}u}{P_n(u)} \tag{1}$$

where is $p_n(u)$ an n-th order polynomial, we can derive the classification of all solutions to the right integral in Eq.(1) using complete discrimination system for the n-th order polynomial.

In this paper, we consider the following the (2 +1)-dimensional nonlinear KdV system:

$$q_t+q_{xxx}-3(qr)_x=0, \tag{2}$$

$$q_x=r_y\ . \tag{3}$$

We reduce the (2 +1)-dimensional nonlinear KdV system to an integrable ODE, and furthermore use complete discrimination system for polynomial to obtain the classification of all single travelling wave atom solutions.

CLASSIFICATIONS

Taking the travelling wave transformation $q=q(\xi)$, $r=r(\xi)$, $\xi=kx+ly+\omega t$ the system is reduced to the following ordinary differential form :

$$\omega q'+k^3q'''-3k(qr)'=0\ , \tag{4}$$

$$k\ q'=lr'\ . \tag{5}$$

By integrating Eq.(4) and Eq.(5) once, we have

$$\omega q+k^3q''-3kqr=c_1, \tag{6}$$

$$kq=lr+c_2, \tag{7}$$

Where c_1 and c_2 are two arbitrary constants.

Moreover from Eq.(7) we can obtain

$$r = \frac{kq - c_2}{l}. \qquad (8)$$

Substituting the expression of r into Eq.(6), we have

$$q'' = \frac{3}{kl}q^2 - (\frac{\omega}{k^3} + \frac{3c_2}{k^2 l})q + c_1, \qquad (9)$$

and integrating Eq.(9) once, then it is

$$\pm(\xi - \xi_0) = \int \frac{\mathrm{d}q}{\sqrt{a_3 q^3 + a_2 q^2 + a_1 q + a_0}}, \qquad (10)$$

where $a_3 = \frac{2}{kl}, a_2 = -(\frac{\omega}{k^3} + \frac{3c_2}{k^2 l}), a_1 = 2c_2, \ a_0 = c_0$.

The corresponding integral form becomes

$$\pm(a_3)^{\frac{1}{3}}(\xi - \xi_0) = \int \frac{\mathrm{d}w}{\sqrt{w^3 + d_2 w^2 + d_1 w + d_0}}, \qquad (11)$$

where $d_2 = a_2(a_3)^{-\frac{2}{3}}$, $d_1 = a_1(a_3)^{-\frac{1}{3}}$, $d_0 = a_0$, ξ_0 is an integral constant.

Denote $F(w) = w^3 + d_2 w^2 + d_1 w + d_0$, the complete discrimination system for $F(w)$ is $\Delta = -27\left(\frac{2d_2^3}{27} + d_0 - \frac{d_1 d_2}{3}\right)^2 - 4\left(d_1 - \frac{d_2^2}{3}\right)^3$, $D_1 = d_1 - \frac{d_2^2}{3}$.

By the complete discrimination system for polynomial, the classifications of all the single traveling wave solutions to the integral formula (11) can be given as follows:

Case 1 : If $\Delta = 0$, $D_1 < 0$, then we have $F(w) = (w-\alpha)^2(w-\beta)$, $\alpha \neq \beta$. The Solutions to q and r can be given by

$$q_1 = \left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left\{(\alpha-\beta)\tanh^2[\frac{\sqrt{\alpha-\beta}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0)]+\beta\right\} \quad (\alpha > \beta) \qquad (12)$$

$$r_1 = \frac{k}{l}\left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left\{(\alpha-\beta)\tanh^2[\frac{\sqrt{\alpha-\beta}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0)]+\beta\right\} - \frac{c_2}{l} \quad (\alpha > \beta) \qquad (13)$$

$$q_2 = \left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left\{(\alpha-\beta)\coth^2[\frac{\sqrt{\alpha-\beta}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0)]+\beta\right\} \quad (\alpha > \beta) \qquad (14)$$

$$r_2 = \frac{k}{l}\left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left\{(\alpha-\beta)\coth^2[\frac{\sqrt{\alpha-\beta}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0)]+\beta\right\} - \frac{c_2}{l} \quad (\alpha > \beta) \qquad (15)$$

$$q_3 = \left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left\{(\beta-\alpha)\tan^2[\frac{\sqrt{\beta-\alpha}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0)]+\beta\right\} \quad (\alpha < \beta) \qquad (16)$$

$$r_3 = \frac{k}{l}\left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left\{(\beta-\alpha)\tan^2\left[\frac{\sqrt{\beta-\alpha}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0)\right]+\beta\right\}-\frac{c_2}{l}\quad(\alpha<\beta) \tag{17}$$

Case 2 : If $\Delta=0$, $D_1=0$, then we have $F(w)=(w-\alpha)^3$. The solutions to q and r are

$$q_4 = \left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left[\left(\frac{2}{kl}\right)^{-\frac{2}{3}}\frac{4}{(kx+ly+\omega t-\xi_0)^2}+\alpha\right], \tag{18}$$

$$r_4 = \frac{k}{l}\left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left[\left(\frac{2}{kl}\right)^{-\frac{2}{3}}\frac{4}{(kx+ly+\omega t-\xi_0)^2}+\alpha\right]-\frac{c_2}{l}. \tag{19}$$

Case 3: If $\Delta>0$, $D_1<0$, then $F(w)=(w-\alpha)(w-\beta)(w-\gamma)$. Suppose $\alpha<\beta<\gamma$, when $\alpha<w<\beta$, we have the solutions to q and r

$$q_5 = \left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left[\alpha+(\beta-\alpha)\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0),k\right)\right], \tag{20}$$

$$r_5 = \frac{k}{l}\left(\frac{2}{kl}\right)^{-\frac{1}{3}}\left[\alpha+(\beta-\alpha)\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0),k\right)\right]-\frac{c_2}{l}. \tag{21}$$

When $w>\gamma$, the solutions are

$$q_6 = \left(\frac{2}{kl}\right)^{-\frac{1}{3}}\frac{\left[-\beta\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0),k\right)+\gamma\right]}{\mathrm{cn}^2\left[\frac{\sqrt{\gamma-\alpha}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0),k\right]}, \tag{22}$$

$$r_6 = \frac{k}{l}\left(\frac{2}{kl}\right)^{-\frac{1}{3}}\frac{\left[-\beta\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0),k\right)+\gamma\right]}{\mathrm{cn}^2\left[\frac{\sqrt{\gamma-\alpha}}{2}\left(\frac{2}{kl}\right)^{\frac{1}{3}}(kx+ly+\omega t-\xi_0),k\right]}-\frac{c_2}{l}, \tag{23}$$

here $k^2=\dfrac{\beta-\alpha}{\gamma-\alpha}$.

Case 4. If $\Delta<0$, then we have $F(w)=(w-\alpha)(w^2+pw+q)$, and $p^2-4q<0$. when $w>\alpha$, the solutions can be given by

$$q_7 = \left(\frac{2}{kl}\right)^{-\frac{1}{3}} \left\{ \alpha + \frac{2\sqrt{\alpha^2 + p\alpha + q}}{1 + \mathrm{cn}\left[(\alpha^2 + p\alpha + q)^{\frac{1}{4}} \left(\frac{2}{kl}\right)^{-\frac{1}{3}} (kx + ly + \omega t - \xi_0), k \right]} - \sqrt{\alpha^2 + p\alpha + q} \right\} \quad (24)$$

$$r_7 = \frac{k}{l}\left(\frac{2}{kl}\right)^{-\frac{1}{3}} \left\{ \alpha + \frac{2\sqrt{\alpha^2 + p\alpha + q}}{1 + \mathrm{cn}\left[(\alpha^2 + p\alpha + q)^{\frac{1}{4}} \left(\frac{2}{kl}\right)^{-\frac{1}{3}} (kx + ly + \omega t - \xi_0), k \right]} - \sqrt{\alpha^2 + p\alpha + q} \right\} - \frac{c_2}{l} \quad (25)$$

here $k^2 = \frac{1}{2}\left(1 - \frac{\alpha + \frac{p}{2}}{\sqrt{\alpha^2 + p\alpha + q}}\right)$.

CONCLUSION

By means of the complete discrimination system for polynomial, we obtain the classifications of all single travelling wave atom solutions to the (2 +1)-dimensional nonlinear KdV system. The solutions are very rich.

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