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Research Article

Optimum Design of Weighting Matrices in Integrated Structural and Control Optimization

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Abstract: Large size, light weight and ease of assembly are some of the desirable attributes in design of space structures. The compromise between a large size and low weight results in a structure that is very flexible, but it makes the control of the structure and its components very difficult. Because these structures are large and flexible, they are very sensitive to environmental effects. Further, these structures possess inherently low damping. Therefore, active control schemes are needed to quickly bring the structure back to its equilibrium position when it is subjected to a disturbance. The purpose of control is to damp out structural vibrations to initial excitations. Linear quadratic regulator (LQR) control method is used for control system design in this paper. Though a majority of the work on integrated structure and control design uses a linear quadratic regulator (LQR) for controller design, the influence of state and control weighting matrices on controller performance is ignored. It is proposed herein that the performance of the control system can be improved by selecting optimum values of the cross sectional areas of the members as well as the entries of the state and control weighting matrices used in the LQR design.
Keywords: Linear quadratic regulator (LQR), damping, matrices.

INTRODUCTION

Because of the strong interaction between the structural design and control system design in active vibration control, simultaneous optimal design of both systems is necessary. The conventional structural and control design approach treats the two designs separately. Each design is optimized based on its objective function but the combined system is not optimal. It is therefore necessary to solve the system in such a way that the structure satisfies the requirements of weight, control effort and performance. This can be done by simultaneous optimization of control and structure. In this method either, the structure and control are linked through constraints related to control and structure design or both the structure and control objective functions are combined as a single cost function. In the work presented in this paper, the structure and control objective functions are linked through constraints related to control and structure.

A significant amount of research has been done on developing methods for the simultaneous design of structure and control system. Fonseca and Bainum [1] proposed two approaches, combined and sequential integrated, to solve the simultaneous structural/control optimization problem. The combined approach uses a cost function that includes both control and structure design considerations whereas the sequential integrated approach uses two separate cost functions for control and structure, but they are matched through constraints. Both approaches yield very similar transient performance in terms of response time and control efforts. Khot *et al.* [2] use weight minimization of the structure as objective function with constraints on the distribution of the eigenvalues and /or damping ratio of the closed loop system. Lee [3] presented a similar approach but considered a control objective. For control design, the most commonly used method is the linear quadratic regulator (LQR). Since the weighting matrices in LQR directly affect the optimal control performance, some discussions have been done for optimal selection of these matrices. Sunar and Rao [4] proposed a methodology for selecting the state and input weighting matrices, *[Q]* and *[R]*, when using linear quadratic regulator in the integrated design of structures and controls. The optimal values of *[Q]* and *[R]*, result in minimizing the performance index and reduced control effort. According to the proposed scheme, the performance index is significantly affected by the changes in the diagonal entries of *[Q]* and *[R]*, matrices so the diagonal entries of *[Q]* and *[R]*, are chosen as design variable to minimize the quadratic performance index. Design was done using a substructure decomposition scheme (for large structures) in order to save the computational cost with very little loss in accuracy. Ohta *et al* [5] have presented a method for selecting weighting matrices in linear quadratic regulator with some diagonal weights that achieve a specified pole location. The proposed method used a polynomial as a desirable pole specification and the weighting matrices are derived in an analytical form. Ochi and Kanai [6] proposed a new way of pole placement by finding a weighting matrix which gives desired locations of the closed loop poles. These poles can then be placed arbitrarily and exactly at the desired positions but does not guarantee the positive definiteness of weighting matrix. The problem of eigen vector assignment is not considered in the paper and the proposed method is computationally expensive.

Choi and Seo [7] presented an LQR design method which has the flexibility of exact eigen structure assignment with stability-robustness properties. The proposed method guarantees that the desired eigen values are assigned exactly and the desired eigen vectors are assigned in the least-square sense. Ang *et al* [8] presented a weighted energy method for selecting the weighting matrices for vibration control of smart composite plates. The quadratic function is selected as a relative measure of strain, kinetic and input energy and their significance is represented through their relative weight factors. The effect of the weight factors on the active modal damping is predicted by modal control method.

Mansouri and Khaloozadeh [9] proposed a genetic approach for an optimal linear quadratic tracking problem. Proper choice of weighting matrices is necessary for satisfying the design specification and this difficulty is overcome by using genetic algorithm. Li *et al* [10] presented a multi-objective evolution algorithm based approach for optimal design of weighting matrices in linear quadratic regulator. By establishing the multi-objective optimization model of LQR, the weighting matrices, *[Q]* and *[R]*, are designed which makes control system meet multiple performance indexes simultaneously. Ghoreishi *et al* [11] carried out a comparative study of different optimization methods for an optimal design of LQR weighting matrices. Closed-loop pole locations, speed of response and maximum level of control effort are combined into an objective function and this multiobjective problem is solved by a weighted sum method and the results for different optimization algorithms are then compared.

Almost all of the referred papers consider the control optimization problem for the optimum selection of the weighting matrices. In this paper the combined structural and control optimization is considered using the structure and control design variables. The overall design of an efficient structural-control system is of interest to both structural and control engineers. Some important aspects of the problem include a minimum weight design, minimizing control energy required and optimum placement of actuators for fast damping of vibrations when the structure is subjected to some external disturbance. In this work the effect of changing the weighting matrices on structural weight and on a controller performance index is studied. The proposed methods results in an improved structural weight and control system performance.

PROBLEM FORMULATION

Controller Design

Control system design requires a mathematical model of the system being controlled. State-space models are commonly used for control system design and are used herein. The starting point for state-space models are the differential equations governing the structural dynamics which are converted into state space form for control system design.

The finite element dynamical equations governing the motion of a controlled structural system are given as:

$$M\ddot{x} + C\dot{x} + Kx = Df \qquad (1)$$

where x is a n x 1 vector of physical coordinates, f is mx1 control vector, *[M]*, *[C]* and *[K]* are n x n mass, damping and stiffness matrices respectively. *[D]* is the n x m applied force distribution matrix which relates the input control force to coordinate system. For forces applied by the actuators acting along the members of the structure, *[D]* is calculated using direction cosines of the constituent members.

Using the coordinate transformation $x = \phi y$, Eq. (1) can be represented in state space form as:

$$\dot{u} = Au + Bf \qquad (2)$$

where y is the vector of modal coordinates, $u = [[y],[\dot{y}]]^T$, is $2n$ x 1 state variable vector, [ϕ] is n x n modal matrix, *[A]* is $2n$ x $2n$ plant matrix and *[B] is* $2n$ x m input matrix.
The plant matrix *[A]* and input matrix *[B]* in Eq. (2) are given as:

$$[A] = \begin{bmatrix} 0 & I \\ -\omega_i^2 & -2\xi_i\omega_i \end{bmatrix} \qquad (3)$$

$$[B] = \begin{bmatrix} 0 \\ \phi^T D \end{bmatrix} \qquad (4)$$

where ξ_i and ω_i denote the damping factor and natural frequency of the i^{th} mode, respectively.

A controller for the system governed by Eq. (2) is designed using linear quadratic regulator (LQR) theory. The optimum control force f is selected to minimize the quadratic performance index, *PI*, which is a compromise between minimum control energy and minimum error requirements, and is defined as:

$$PI = \int_0^\infty (u^T[Q]u + f^T[R]f)dt \qquad (5)$$

where *[Q]* is a positive semi definite state weighting matrix and *[R]* is a positive definite control weighting matrix. The optimum feedback control law is given as $f = -[\kappa]u$ where $[\kappa]$ is the feedback gain matrix defined as $[\kappa] = [R]^{-1}[B]^T[P]$, and *[P]* is the solution to matrix Riccati equation:

$$[A]^T[P] + [P][A] + [Q] - [P][B][R]^{-1}[B]^T[P] = [0] \qquad (6)$$

The minimum value of the quadratic performance index is given as:

$$PI^* = u^T(0)[P]u(0) \qquad (7)$$

where *u(0)* is the initial state vector. It has been found that the expected value of PI^* over a set of possible initial states *u(0)* is equivalent to trace of *[P]*. It has been shown [12] that the minimization of the quadratic control effort is proportional to trace *[P]*, therefore trace *[P]* is considered as objective function in the present work.

Substituting $f = -[\kappa]u$ in Eq. (2) yields:

$$\dot{u} = ([A] - [B][\kappa])u = [A_{cl}]u \qquad (8)$$

The eigenvalues of the closed loop matrix $[A_{cl}]$ are a set of complex conjugate pairs given as:

$$\lambda_i = \alpha_i \pm j\beta_i \qquad i=1,2,\ldots n \qquad (9)$$

where $j = \sqrt{-1}$ and $|\lambda_i| = \sqrt{\alpha_i^2 + \beta_i^2}$. The closed loop damping ratio ξ_i associated with λ_i is given as

$$\xi_i = -\frac{\alpha_i}{\sqrt{\alpha_i^2 + \beta_i^2}} \qquad i=1,2,\ldots n \qquad (10)$$

The solution to Eq. (8) for a given initial condition $u(0)$, is given as:

$$u(t) = e^{[A_{cl}]t}u(0) \qquad (11)$$

Equation (11) can be used to find the dynamic response of the structure when it is subjected to some initial disturbance $u(0)$. The MATLAB function ode45 can be used to solve the first order differential equation given in Eq. (8).

SOLUTION PROCEDURE

The first problem considered herein involves solving a simultaneous structural and control design problem for minimization of trace$[P]$ by fixing the actuators at some specific locations and fixing the $[Q]$ and $[R]$ matrices (see Eq. 5) as identity matrices. Next the effect of changing the $[Q]$ and $[R]$ matrices is studied. Two cases are considered: (i) the cross-sectional areas of members are fixed and $[Q]$ and $[R]$ matrices are varied; (ii) the member cross-sectional areas as well as entries of $[Q]$ and $[R]$ matrices are varied. The optimization procedure is such that an initial design variable vector is selected, based on that the mass, stiffness and damping matrices are assembled and the structural frequencies and mode shapes are calculated. Using the structural frequencies and mode shapes, plant matrix *[A]* and input matrix *[B]* are assembled. *[D]* matrix is then set up based on the location of actuators. LQR problem is solved next and optimum gain and Riccati matrix solution is found. The objective function and all constraints values are calculated. The design variables are updated and the solution is repeated until no improvement in objective function is possible.

INFLUENCE OF WEIGHTING MATRICES ON OPTIMUM DESIGN

The effect of changing the weighting matrices is presented in this section with actuators fixed at some specific locations. Two cases are considered. In the first case, the weighting matrices are assumed to be fixed and cross-sectional areas are varied to optimize the controller performance index. The second case involves varying both the cross-sectional areas and weighting matrices to optimize the controller performance index.

Baseline Design — Weighting Matrices fixed

A design example is presented next for studying the effect of using optimum values for weighting matrices on the optimum design of structure. Towards this end, a baseline design is established first. In this design, only member cross-sectional areas are varied to optimize the controller performance index, the weighting matrices are assumed to be fixed.

Design Example

The 12-member ACOSS four structure is shown in Fig. 1 [12]. This structure, designed by Draper Labs, is the simplest non-planar geometry representing a large space structure. All physical and geometric properties of the structure are nondimensionalized. The boxed numbers (in Fig. 1) represent the nodes while the others represent the elements. The edges of the truss consist of six elements (1 through 6) of length 10 units each and six bipod legs (7 through 12) of $2\sqrt{2}$ units each. The nodal coordinates of the system are given in Table 1. The structure has twelve degrees of freedom, three at each of the four free nodes. The Young's modulus of the members is taken as 1.0 and the weight density of the material is assumed to be 0.001. The size of $[Q]$ matrix is $2n \times 2n$ and $[R]$ matrix is $m \times m$ and they are assumed to be identity matrices where n denote the degrees of freedom and m denote the number of actuators present. The values of n and m here are 12 and 6 respectively. The cross-sectional areas of the members are treated as design variables. A total of six actuators are present in elements 7 through 12.

Table 1: Nodal Coordinates of Acoss Four

Node	X	Y	Z
1	0	0	10.165
2	-5	-2.887	2
3	5	-2.887	2
4	0	5.7735	2
5	-6	-1.1547	0
6	-4	-4.6188	0
7	4	-4.6188	0
8	6	-1.1547	0
9	-2	5.7735	0
10	2	5.7735	0

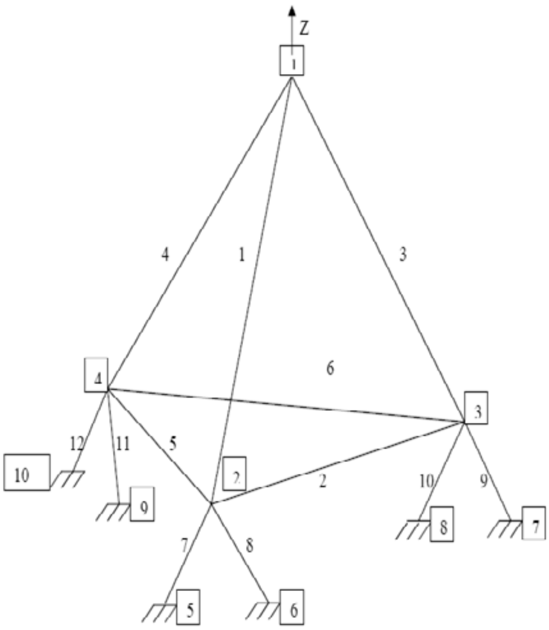

Fig-1: ACOSS FOUR Structure

The dynamic response of the structure to an initial disturbance is also studied by measuring the displacement associated with the line of sight (LOS). Node 1 represents the antenna feed, and its motion measures the deviation from the LOS. The dynamic response of the optimum structure is initiated by a unit displacement at node 2 in the x-direction at t=0.

Optimization Problem Formulation

A minimization of the controller performance index (trace$[P]$) is considered as the objective function with the cross-sectional areas of the elements of the structure as design variables. Mathematically, the optimization formulation is stated as:

Minimize trace$[P]$

by varying A_i

subject to

$$\begin{aligned} &0.16434 - \xi_1 \leq 0 \\ &1.3374 - \beta_1 \leq 0 \\ &1.5 - \beta_2 \leq 0 \\ &10 \leq A_i \leq 2000 \quad i = 1, \ldots\ldots 12 \end{aligned} \tag{12}$$

The optimization problem is solved using the Method of Feasible Directions and the solution steps are outlined in Fig. 2.

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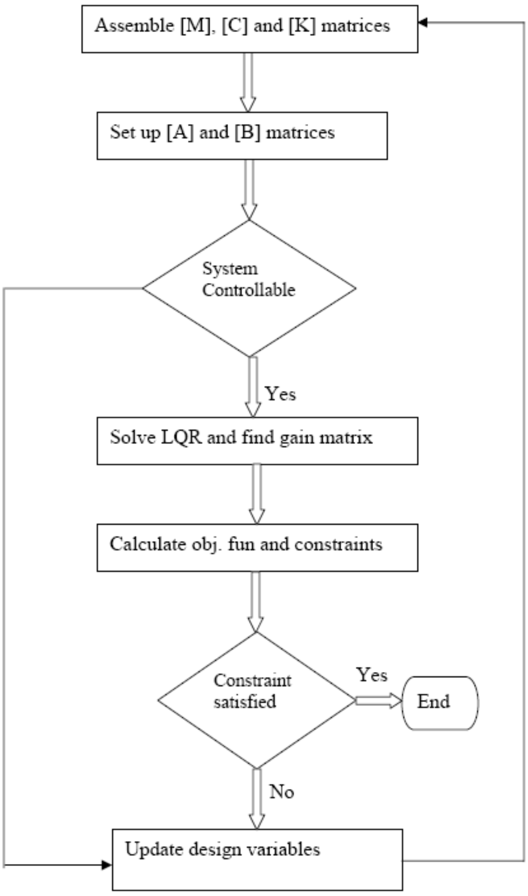

Fig-2: Steps in the optimization process

RESULTS

The starting values of the cross-sectional areas, closed-loop damping ratios, closed-loop eigenvalues and square of the natural frequencies are given in Table 2. The value of the weight at this starting design is 43.69 and trace $[P]$ is 1763.2. The LOS error for the transient response is given in Fig. 3. The transient response is simulated by finding the solution to Eq. (11) for 60 seconds at 0.05 seconds time intervals. The magnitude of LOS error is calculated at each interval.

Table 2: Nominal Areas, closed loop damping ratio, closed loop eigenvalues and squares of natural frequencies

Areas	Damping Ratio	Real Part	Imag. Part	Sq. of natural Frequencies
1000	0.0548	-0.0734	1.3375	1.79
1000	0.0655	-0.1088	1.6573	2.75
100	0.0738	-0.2121	2.8674	8.26
100	0.0802	-0.2357	2.9302	8.63
1000	0.084	-0.2837	3.3664	11.4
1000	0.0864	-0.362	4.1732	17.53
100	0.0761	-0.3536	4.6332	21.58
100	0.0723	-0.3421	4.72	22.39
100	0.0341	-0.2901	8.4986	72.31
100	0.0298	-0.2742	9.2062	84.83
100	0.0207	-0.2126	10.2456	105.02
100	0.0064	-0.0823	12.8504	165.14

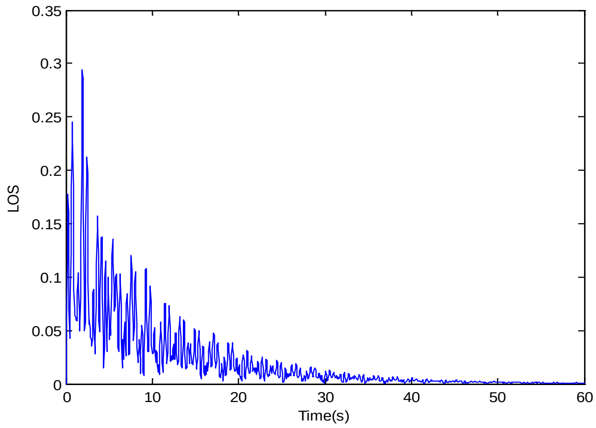

Fig-3: Transient response of structure at nominal design

Using the nominal values of the areas as starting design for the optimization problem, the optimum values of the cross-sectional areas, closed-loop damping ratios and closed-loop eigenvalues are given in Table 3. The optimum trace $[P]$ is 715 and the weight of the structure at this design is 22.9. A 60% reduction in trace $[P]$ and 48% reduction in weight is obtained at the optimum design. The LOS error at the optimum solution is 1.52 and is shown in Fig. 4.

Table 3: Optimum Areas, closed loop damping ratio, closed loop eigenvalues and squares of natural frequencies

Areas	Damping Ratio	Real Part	Imag. Part	Square of natural frequencies
430.09	0.1635	-0.2218	1.336	1.79
424.82	0.0921	-0.0769	1.4926	2.25
306.03	0.0963	-0.2073	2.5533	6.57
397.06	0.0878	-0.197	2.8917	8.41
293.22	0.0655	-0.207	3.7632	14.21
222.21	0.0662	-0.2852	4.3519	19.02
122.85	0.0519	-0.2472	5.2807	27.94
304.48	0.0514	-0.3113	5.6312	31.79
27.89	0.0451	-0.3465	6.1208	37.56
50.53	0.0398	-0.2702	7.0125	49.25
142.49	0.0347	-0.315	8.008	64.17
120.54	0.0272	-0.2583	8.8766	78.81

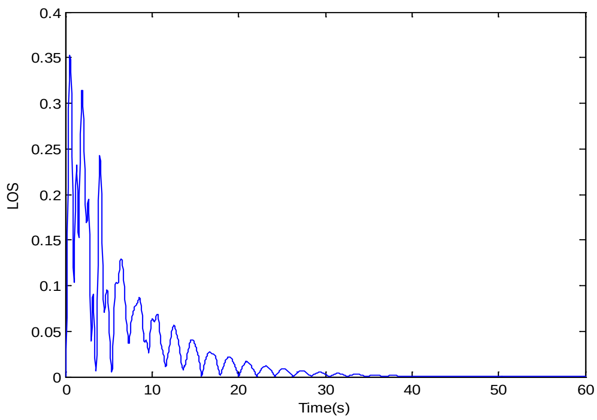

Fig-4: Transient response of structure at optimum design

When comparing the nominal and optimum designs, it is seen that in the case of nominal design, the frequencies associated with modes 3 and 4 and modes 7 and 8 are close to each other. However, in the case of optimum design, the frequencies are spread out and no two frequency values are as close as in the nominal design case.

Effect of Changing the Weighting Matrices

In order to see the effect of changing the weighting matrices on the controller performance, the same ACOSS four structure (Fig.1) is considered for the optimization problem. A minimization of trace$[P]$ is considered as the objective function with diagonal entries of the state and control weighting matrices, $[Q]$ and$[R]$, treated as design variables. The design constraints imposed on the problem are given by Eq. (12) with one additional constraint that all the diagonal terms of $[Q]$ and $[R]$ matrices should be greater than or equal to 1. The controls toolbox in Matlab is used for solving Riccati equation and for finding the control gains used in the LQR control method.

Results

Two scenarios are considered next for studying the effect of varying weighting matrices on the optimum controller performance with (i) member cross-sectional areas at fixed values and (ii) optimum values determined for member cross-sectional areas.

Areas fixed at nominal values

The only problem variables are entries of $[Q]$ and $[R]$ matrices. Two different starting designs are considered. When starting value of $[Q]$ and $[R]$ are taken as $[I]$, where $[I]$ is an identity matrix, the minimum trace $[P]$ is found to be 1843.06. The optimum values of entries of $[Q]$ matrix are: Q_1 =13.5 and Q_{13} =7.05 All others $[Q]$ values are at the lower bound which is 1.0. All optimum $[R]$ values converge to the lower bound of 1.0. The second starting design used the value $[Q]$ =10$[I]$ and $[R]$=$[I]$. In this case, the minimum value of trace$[P]$ is 1844.86 and only Q_1 =25.06 and rest of $[Q]$ values are all at 1.0. Also all entries of $[R]$ matrix are 1.0 at the optimum solution. Some other starting points are also considered and they are shown in Table 4 with the corresponding weights, trace $[P]$ and LOS error values. It can be seen from Table 4, different values for the starting design results in different values for the optimum design variables. This indicates there are several local optima and the results are not globally optimum.

Table 4: Areas fixed at nominal values

	Starting point				
	Q=R=I	Q=R=10I	Q=10I, R=I	Q*	Q*
				R_{1-3}=10, R_{4-6}=1	R_{1-3}=1, R_{4-6}=10
Weight	43.70	43.70	43.70	43.70	43.70
trace $[P]$	1843.06	1843.60	1844.86	1853.82	1843.97
LOS	1.055	1.055	1.055	1.052	1.055
Optimum Q	Q_1=13.5	Q_1=24.39	Q_1=25.06	Q_1=30.05	Q_1=21.98
	Q_{13}=7.05				Q_{13}=2.47

Q^* = Q_{1-8}=1, Q_{9-16} =10 and Q_{17-24} =5

Areas and Q and R Matrices as design variables

In order to improve upon the results reported in the previous section, the optimization problem is solved by considering the member cross-sectional areas and the diagonal entries of [Q] and [R] as design variables. The design variables in this case are 42 (12 cross sectional areas, 24 diagonal entries of [Q] and 6 diagonal entries of [R]). At the starting design of [Q]=[R]=[*I*], the optimum value of trace $[P]$ is 553.46 with optimum Q_1 =3.21 and Q_{13} =3.54, all other Q's and R's converge to lower bound of 1.0. The weight of the structure at this optimum design is 15.2 and the LOS error is 1.88.

By changing the starting design as [Q]=10[*I*] and $[R]$=$[I]$, optimum value of trace $[P]$=550.06 with optimum Q_1 =7.47 all other Q's and R's converge at 1. The weight of the structure is 15.14 and the LOS error is 1.52 and is shown in Fig. 5. Some other starting points are also considered and they are shown in Table 5 with the corresponding weights, trace [P] and LOS error values. The optimum values of the cross- sectional areas are given in Table 6. It can be seen from the results presented herein that a 34% reduction in weight and 23% reduction in trace $[P]$ can be achieved by considering the member cross-sectional areas and diagonal entries of $[Q]$ and $[R]$ matrices as design variables. Therefore in order to improve the overall performance of structure, member cross-sectional areas along with entries of $[Q]$ and $[R]$ matrices should be considered as design variables.

Table 5: Areas and diagonal [Q] and [R] as design variables

Starting point	Q=R=I	Q=R=10I	Q=10I, R=I	Q^*
				R_{1-3}=1, R_{4-6}=10
Weight	15.23	15.13	15.14	15.29
trace [P]	553.46	550.24	550.06	554.92
LOS	1.88	1.89	1.52	1.87
Optimum Q	Q_1=3.21	Q_1=7.83	Q_1=7.47	Q_1=1.47
	Q_{13}=3.54			Q_{13}=4.46

Q^* = Q_{1-8}=1, Q_{9-16} =10 and Q_{17-24} =5

Table 6: Optimum cross-sectional areas

Element	Q=R=I	Q=R=10I	Q=10I, R=I	Q^*
				R_{1-3}=1,R_{4-6}=10
1	271.58	310.42	235.8	277.45
2	209.78	192.14	247.74	211.33
3	205	208.39	240.68	205.6
4	217.4	201.48	216.41	216.07
5	220.88	202.78	217.91	219.72
6	228.03	232.24	182.6	228.89
7	66.16	160.95	195.33	67.28
8	187.68	76.75	84.15	185.8
9	107.37	50.51	91.89	105.79
10	96.99	99.4	70.11	96.22
11	53.09	107.61	122.31	55.04
12	93.64	90.56	50.17	93.09

Q^* = Q_{1-8}=1, Q_{9-16} =10 and Q_{17-24} =5

Table 7: Weight and trace [P] values for three cases

	[Q] and[R] design variables, Areas fixed	Areas design variable, [Q]=[R]=[*I*]	Area and [Q], [R] as design variables
Trace [P]	1843	715	550
Weight	43.69	22.9	15.14

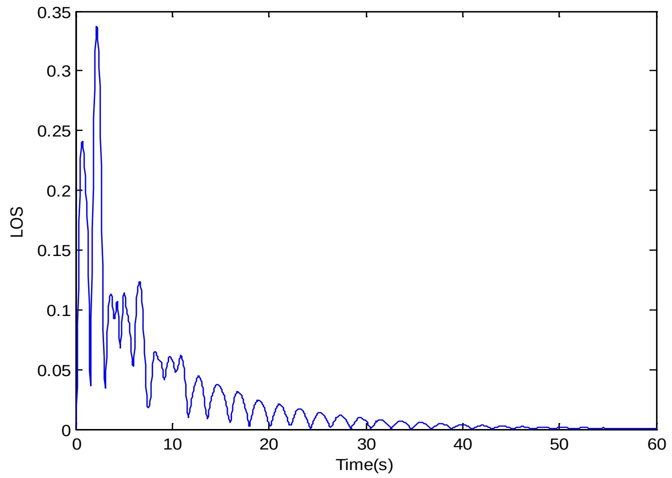

Fig-5: Response when areas and [Q] and [R] are varied

CONCLUSION

This paper presents an approach to integrated structural and control optimization for selecting optimum weighting matrices and cross sectional areas of the structure. The overall design of an efficient structural-control system is of interest to both structural and control engineers. For an efficient controller, trace [P] is minimized by treating the weighting matrices as design variables. The natural frequencies of the controlled system and time required to damp out the vibrations are specified by putting constraints on closed-loop eigenvalues and damping ratios. From a structural viewpoint, the designer wants to minimize the weight of the structure by varying the cross-sectional areas of the members and ensuring that the stresses in members due to applied loads don't exceed permissible limits. In this work the effect of changing the weighting matrices on structural weight and on a controller performance index is studied. From this work, it is concluded that great savings in the control energy as well as structural weight is possible by using both the cross sectional areas and diagonal entries of weighting matrices as design variables.

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