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Free Undamped Motion in Spring Mass System

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Abstract **Review Article**

Assume that a mass m is fastened to the free end of a flexible spring that is hanging vertically from a rigid support [5]. Naturally, the mass of the spring will determine how much it stretches or elongates; different weight masses will stretch the spring in different ways. The spring itself exerts a restoring force F that is proportional to the amount of elongation s and opposed to the direction of elongation, according to Hooke's law. Put simply. When a mass m is connected to a spring, the mass extends the spring by a certain amount, reaching an equilibrium where the restoring force ks balances the mass W. Remember that [6] defines weight.

$$w = mg \tag{0.3.1}$$

the condition of equilibrium is

$$mg = -ks \tag{0.3.2}$$

or

$$mg - ks = 0 \tag{0.3.3}$$

In the event that the mass deviates from its equilibrium position by an amount x, the spring's restoring force is equal to k(x + s).

We can associate Newton's second rule with the net, or resultant, force of the restoring force and the weight W is balanced by the restoring force ks, provided that there are no retarding forces operating on the system and that[6] the mass vibrates free of other external forces—free motion. Remember that [6] defines weight.

$$\frac{d^2x}{dt^2} = -k(s + x) + mg \qquad (0.3.5)$$

$$= -kx + mg - ks = -kx \qquad (0.3.5)$$

The spring's restoring force works in the opposite direction of motion, as shown by the negative sign. Additionally, we follow the tradition that displacements recorded in positive values below the equilibrium position [6].

Keywords: weight masses, spring, motion.

0.1.1 DE OF FREE UNDAMPED MOTION

By dividing (1) by the mass m, we obtain the second-order differential equation

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \tag{0.3.6}$$

Or

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{0.3.7}$$

This equation describes simple harmonic motion or free undamped motion.
Where

$$\omega^2 = frackm \tag{0.3.8}$$

Two obvious initial conditions associated with

$$x(0) = x_0 \tag{0.3.9}$$

and

$$x(\acute{}0) = x_0 \tag{0.3.10}$$

The initial displacement and initial velocity of the mass, respectively. For example, if

$$x_0 > 0, x_0 < 0 \tag{0.3.11}$$

The mass starts from a point below the equilibrium position with an imparted upward velocity [6]. When

$$x(\acute{}0) = 0 \tag{0.3.12}$$

0.1.2 EQUATION OF MOTION

To solve equation (2) we note that the solutions of its auxiliary equation:

$$m^2 + \omega^2 = 0 \tag{0.3.13}$$

are the complex numbers:

$$m_1 = \omega i \tag{0.3.14}$$

$$m_2 = -\omega i \tag{0.3.15}$$

we find the general solution of

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{0.3.15}$$

to be

$$x(t) = c_1 cos\omega t + c_2 sin\omega t \tag{0.3.17}$$

The period of motion described

$$I = \frac{2\pi}{\omega} \tag{0.3.18}$$

The time it takes the mass to complete one cycle of motion, expressed in seconds, is represented by the number T. One full oscillation of the mass is called a cycle.

$$I = \frac{2\pi}{\omega} \tag{0.3.19}$$

The frequency of motion is

$$t = \frac{1}{T} = \frac{\omega}{2\pi} \tag{0.3.20}$$

and is the number of cycles completed each second.

The number

$$\omega = \frac{rk}{m} \tag{0.3.21}$$

(expressed in radians per second) is the system's circular frequency. Whichever text you choose to read, both

$$f = \omega/2\pi and \omega \quad (0.3.22)$$

are also referred to as the natural frequency of the system.

0.1.3 FREE DAMPED MOTION

Assume that this force is provided by a constant multiple of dx for the duration of the discussion that follows. This derives from Newton's second law when the system is not subjected to any additional external pressures [5].

$$m\frac{d^2x}{dt^2} = -kx - \beta\frac{dx}{dt} \quad (0.3.23)$$

Divid this equation by m will get:

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = F(t) \quad (0.3.24)$$

Where:

$$F(t) = \frac{f(t)}{m} \quad (0.3.25)$$

and

$$2\lambda = \frac{\beta}{m} \quad (0.3.26)$$

We have two options for solving the latter nonhomogeneous equation: changing the parameters or using the approach of undetermined coefficients.

0.1.4 DRIVEN MOTION

DAMPING AND DRIVEN MOTION DETECTION Let us now assume an external force f (t) applied to a mass on a spring that is vibrating. For instance, f (t) can stand for a driving force that causes the spring support to oscillate vertically.

0.1.5 TRANSIENT AND STEADY-STATE TERMS

When F is a periodic function, such as

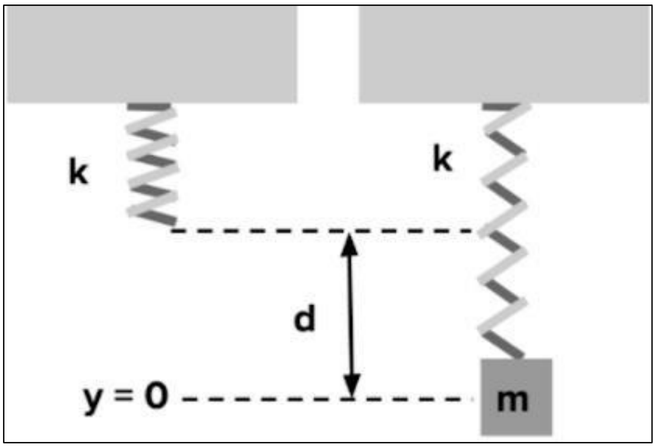
$$F(t) = F_0 \sin \gamma t \quad (0.3.27)$$

$$Or\ F(t) = F_0 \cos \gamma t \quad (0.3.28)$$

the general solution of for $\lambda > 0$

0.1.6 DE OF DRIVEN MOTION WITHOUT DAMPING

In any oscillatory mechanical system, a periodic impressed force with a frequency close to or equal to the frequency of free undamped vibrations can be extremely problematic in the absence of a damping force [5].



RESULT AND DISCUSSION

An oscillatory system that is subjected to damping experiences an influence that lessens, restricts, or eliminates oscillations. This work proposes a one-step sixth-order computing approach for solving second order free damped and undamped motions in mass-spring systems. A continuous computational hybrid linear multistep technique was created by using the interpolation and collocation of power series approximate solution. This method was then evaluated at grid locations to provide a continuous block method. When the continuous block technique was assessed at particular grid points, the resulting discrete block method was obtained. Investigations into the method's fundamental characteristics revealed that it was zero-stable, consistent, and convergent.

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