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Circling the Regular Hexagon with Straightedge and Compass in Euclidean Geometry

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Abstract **Review Article**

The topic of this article means "one can construct a circle (O, r), of which area is equal exactly to a given regular hexagon, using a straightedge and a compass only".
No scientific theory lasts forever, but specific research and discoveries continuously build upon each other. The three classic ancient Greek mathematical challenges likely referring to are "Doubling The Circle", "Trisecting An Angle" and "Squaring The Circle", all famously proven Impossible under strict compass-and-straightedge constraints, by Pierre Wantzel (1837) using field theory and algebraic methods, then also by Ferdinand von Lindemann (1882) after proving π is transcendental. These original Greek challenges remain impossible under classical rules since their proofs rely on deep algebraic/transcendental properties settled in the 19th century. Recent claims may involve reinterpretations or unrelated advances but do overturn these conclusions above. Among these, the "Squaring The Circle" problem and related problems involving π have captivated both professional and amateur mathematicians for millennia. The title of this paper refers to the concept of "constructing a circle that has the exact area of a given regular hexagon," or "Circling The Regular Hexagon", for short. This research idea arose after the "Squaring The Circle" problem was studied and solved and published in "SJPMS" in 2024 [7]. This paper presents an exact solution to constructing a circle that is concentric with and has the same area as a given regular hexagon. The solution does not rely on the number π and adheres strictly to the constraints of Euclidean Geometry, using only a straightedge and compass. The technique of "ANALYSIS" is employed to solve this "Circling The Regular Hexagon" problem precisely and exactly with only a straightedge and compass, without altering any premise of the problem. This independent research demonstrates the solution to the challenge using only these tools. All mathematical tools and propositions in this solution are derived from Euclidean geometry. The methodology involves geometric methods to arrange the given circle and its equal-area regular hexagon into a concentric position. Building on this method of exact "Circling The Regular Hexagon," one can deduce an equivalent problem to formulate a new mathematical challenge: "Regular Heptagoning The Circle" (i.e., constructing a regular heptagon that has the same area as a given circle, using only a straightedge and compass).
Keywords: Circling the regular hexagon, Circle; regular hexagoning circle; circle hexagonise; Circle area of an hexagon; constructing a circle with the same area as a hexagon; Euclidean geometry; straightedge and compass.

INTRODUCTION

For over three millennia, three classical geometric well-known problems from ancient Greece—Squaring the Circle, Doubling the Cube, and Trisecting an Angle— emerged and have challenged the ingenuity of mathematicians. These problems, first posed with the constraint of using only an unmarked straightedge and compass, have long been declared impossible to solve within the framework of Euclidean geometry. Foundational impossibility proofs by 19th-century mathematicians such as Pierre Wantzel and Ferdinand von Lindemann—using algebraic field theory and transcendental number theory—seemed to confirm that exact solutions were unattainable. Specifically, the transcendence of π implied that a square with an area exactly equal to that of a given circle could not be constructed through purely geometric means. Yet, while these algebraic impossibility theorems are rigorous within their own domains, they diverge in nature from the original geometric spirit of the ancient challenges. The historical emphasis on purely constructive geometry, devoid of algebra, arithmetic or trigonometry, leaves space for a critical re-evaluation.

Among them, Hippocrates meticulously studied the problem of squaring the circle. The problem is stated as follows: Using only a straightedge and a compass, is it possible to construct a square with an area equal to a given circle? These problems were proven unsolvable using only straightedge and compass in the 19th century [4]. In 1882, mathematician Ferdinand von Lindemann proved that π is an irrational number, which means that it is impossible to construct a square with the same area as a given circle using only a straightedge and a compass, as posed by Hippocrates. One of the most fascinating aspects of this problem is that it has captured the interest of mathematicians throughout the history of mathematics. From the earliest mathematical documents to today's mathematics, the problem and its relation to π have intrigued both professional and amateur mathematicians for millennia.

A significant step forward in proving the impossibility of squaring the circle occurred in 1761 when Lambert proved that π is irrational [2] & [3]. This, however, was not sufficient to demonstrate the impossibility of squaring the circle using a straightedge and compass, as certain algebraic numbers can be constructed with these tools. Despite the proof of the impossibility of squaring the circle, the problem has continued to captivate mathematicians and the general public alike, remaining an important topic in the history and philosophy of mathematics.

In 1837, French mathematician L. Wantzel proved that the three classical problems of ancient Greece are impossible to solve using only a straightedge and compass. Although based on algebra, Pierre Laurent Wantzel (1837) declared the verdict, "Arbitrary angle trisection is impossible", I experienced that my achieved procedure was possibly not arrested by this verdict [4]. After applying my method to trisect an arbitrary angle, I hope readers can trust algebra cannot affect the successful geometrical constructions of angle trisection.

Therefore, the problems of squaring the circle, doubling the cube, and trisecting an angle have been studied for centuries and remain unsolvable with these tools to this day. As of December 2022, no mathematician has found exact solutions to classical problems such as "Doubling the Cube," "Squaring the Circle," or "Trisecting an Angle" using only a compass and straightedge.

However, in 2023, my research article, published on the SJPMS journal on 31/10/2024 (SJPMS published link: https://doi.org/10.36347/sjpms.2024.v11i10.004*) as an exact solution to the "Squaring the Circle" problem [6], provides a counter-proof to the impossibility stated by Wantzel in 1837, [4].* This article arises from such a re-evaluation and documents a new geometric approach—remaining strictly within the constraints of compass and straightedge—that claims to construct precise solutions to several of these age-old problems. Beginning with a geometric interpretation of Lao Tzu's aphorism "Lao Tzu's aphorism in the Tao Te Ching / Dao De Jing book - "*The Great Tao is simple, very simple / Đại Đạo thì đơn giản, rất giản dị*", - the author explores the power of simplicity and concentric arrangement. Through detailed analysis, an exact method for Squaring the Circle is developed, followed by solutions to Doubling the Cube and Trisecting an Angle, each derived using geometric reasoning alone in 2023, 2024 & 2025 [6], [8] & [9].

In the past, knowledge was often considered scientific if it could be confirmed through specific evidence or experiments. However, Karl Popper, in his book *Logik der Forschung* (The Logic of Scientific Discovery), published in 1934, demonstrated that a fundamental characteristic of scientific hypotheses is their ability to be proven wrong (falsifiability). Anything that cannot be refuted by evidence is temporarily regarded as true until new evidence emerges. For instance, in astronomy, the Big Bang theory is widely accepted, but in the future, anyone who discovers a flaw in this theory will be acknowledged by the entire physics community. Furthermore, no scientific theory lasts forever; rather, it is specific research and discoveries that continually build upon one another [1].

Starting from accepted premises, without proof, one uses deductive reasoning to arrive at theorems and corollaries. With different premises, we develop different mathematical systems. For example, the premise "from a point outside a line, only one parallel line can be drawn to the given line" leads to Euclidean geometry. If we assume that no parallel lines can be drawn from that point, we enter the realm of Riemannian geometry. Alternatively, Lobachevskian geometry assumes that an infinite number of parallel lines can be drawn through that point. It is important to clarify that this does not imply a square with an equal area to a circle does not exist. If the circle has an area A, a square with a side length equal to the square root of A would have the same area. It does not imply that it is impossible to solve the problem using only a straightedge and compass. *Thus, I provided an exact solution to the "Squaring the Circle" problem without altering the premises of Euclidean geometry or the constraints of straightedge and compass [6]. In 2024, I also solved the inverse problem, titled "Circling The Square with Straightedge & Compass in Euclidean Geometry," [7].*

"Circling The Regular Hexagon" refers to the process of "*constructing a circle that has the same area as a given regular hexagon and is concentric with that hexagon*." This concept came to me spontaneously after solving the problem of "circling the square." I thought, "*If one can circle a square then can one also circle a regular hexagon, using only a straightedge and compass in Euclidean Geometry?*" In other words, "Circling The Regular Hexagon" is a new challenge problem that arose from the exact solution presented in my "Circling The Square" paper, published in *SJPMS* [7]. Therefore, the "Circling The Regular Hexagon" challenge and its solution have not existed in the field of Mathematics until it was solved and published by this paper.

In seeking solutions to such problems, geometers developed a special technique called "ANALYSIS." They would assume the problem had been solved, and by investigating the properties of the solution, they would work backward to identify an equivalent problem that could be solved based on the given conditions. To obtain the formally correct solution to the original problem, geometers would then reverse the process: starting with the data to solve the equivalent problem derived through analysis, and then using that solution to solve the original problem. This reversed procedure is known as "SYNTHESIS." Analysis and synthesis are fundamental methods in mathematical reasoning and problem-solving. Analysis involves the process of breaking down complex mathematical problems or structures into simpler, more manageable components. It enables mathematicians to understand underlying principles, identify patterns, and isolate essential variables. Through analytical methods, one can derive conclusions, test hypotheses, and explore the logical consequences of assumptions. Conversely, synthesis entails the combination of previously understood elements to construct more complex systems or to develop general theories. It often follows analysis, using the insights gained to build new models, solve broader problems, or formulate proofs. The interplay between analysis and synthesis fosters a deeper comprehension of mathematical concepts and drives the progression of mathematical theory. Together, these complementary approaches are vital to the advancement of mathematics, underpinning both theoretical inquiry and practical application.

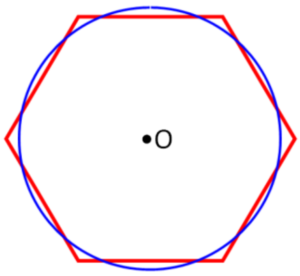
I have adopted both the "ANALYSIS" and "SYNTHESIS" techniques to solve the "Regular Triangling A Circle" problem exactly, using only a straightedge, compass, in Euclidean Geometry, without involving the irrational number π.

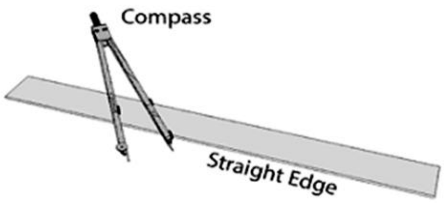
The inspiration for this research arose after the solution to the "Circling The Squaring" problem was published in SJPMS in 2024 [7]. If it is possible to circle a square, using Euclidean geometry, the question arises: *why is it difficult to circle a regular hexagon?* In this context, the given regular hexagon intersects a concentric circle at 12 vertices of an inscribed regular dodecagon of the circle. This ***regular dodecagon*** serves as the solution to the new problem of "Circling The Regular Hexagon." The remaining task is to prove that the area of this circle is exactly equal to the area of the given regular hexagon. This paper also introduces a new mathematical tool, the "Regular Dodecagon Ruler," and provides a proof for the following:

(1) a circle intersects the given regular hexagon at the 12 vertices of a special dodecagon;

(2) the construction process forms a regular dodecagon inscribed in the circle.

This paper's solution to the "Circling The Regular Hexagone" problem naturally leads to the concept of "Circling the regular heptagon," "Circling the regular octagon," and so on.





Part I: PROOFS OF NEW PROPOSITIONS

I.1 Theorem 1:

If there exists a circle that is concentric with a given regular hexagon and has the same area, then its circumference intersects the sides of the hexagon at 12 points, forming a dodecagon inscribed in the circle.

PROOF:

Assume that the blue circle (O, r) in Figure 1 below is the desired shape: it is concentric with a given regular hexagon (shown in red in Figure 1, below) and has the same area as the given hexagon. Then:

- The area of this blue circle (O, r) is obviously less than the area of the circumscribed black circle (O, R) of the regularly given hexagon (red colour in Figure 1 below);
- The area of the blue circle (O, r) is also clearly greater than the area of the inscribed green circle (O, r) within the given hexagon (see the red hexagon in Figure 1 below).

Therefore, this circle (O, r) intersects the six sides of the hexagon at 12 points. These 12 intersection points form the vertices of a dodecagon inscribed in the circle (O, r), as required.

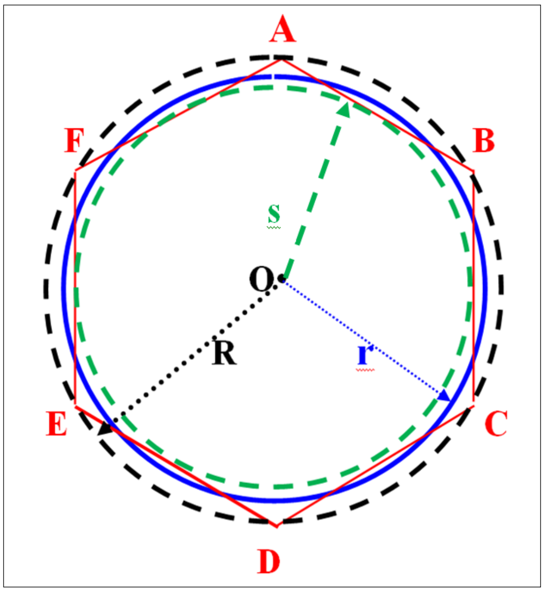


Figure 1: ***The given regular hexagon (red colour), the blue circle (O, r), and the 2 dashed circles (O, s) and (O, R).***

I.2 Theorem 2:

Given a regular hexagon ABCDEF with circumscribed circle (O, R), there exists a circle (O, r) concentric with the hexagon, of which:

1. *The area of (O, r) equals the area of ABCDEF,*
2. *The intersection points of (O, r) with ABCDEF and its symmetrically constructed counterpart A′B′C′D′E′F′ form a regular dodecagon abcdefghijkl, inscribed the resulting circle (O, r).*

PROOF:

Step 1: Area Equivalence and Circle-Hexagon

Configuration

Let ABCDEF be a regular hexagon inscribed in circle (O, R). The hexagon area is A.

Assume there exists a circle (O, r) that has the same area πr^2 as the area A of a given regular hexagon ABCDEF and is concentric to the hexagon (Figure 2 below).

By Theorem 1 (Section I.1 above), circle (O, r) intersects each side of ABCDEF at two points, yielding 12 distinct points a,b,c,d,e,f,g,h,i,j,k,l (Figure 2).

Step 2: Symmetric Construction of Inner Hexagon A′B′C′D′E′F′

Extend chord al of (O, r) to intersect (O, R) at points A′ and F′. Connect O to A′, and draw a line symmetric to A′F′ about OA′, intersecting (O, R) at B′. By symmetry, angle F′A′B′ is bisected by OA', and chord A′B′ in (O, R) overlaps with chord bc in (O, r) in Figure 2, below.

Repeating this symmetric construction from B′ to F′, to obtain a regular hexagon A′B′C′D′E′F′ inscribed in (O, R). Therefore, the hexagon A′B′C′D′E′F′ is also constructible with straightedge and compass. This hexagon is congruent to hexagon ABCDEF and shares the same centre O.

Step 3: Regular Dodecagon Formation

The intersections of ABCDEF and A′B′C′D′E′F′ occur precisely at the 12 points a, b, c, d, e, f, g, h, i, j, k & l. These points lie on (O, r) and are equidistant from O, forming a regular dodecagon. The regularity follows from:

1. The rotational symmetry of ABCDEF and A′B′C′D′E′F′,
2. The equal spacing of intersection points due to overlapping chords in (O, r), in Figure 2, below.
   Therefore, A'B'C'D'E'F' is also the inscribed regular hexagon in the circle (O, R). This shows that the two equivalent regular hexagons ABCDEF & A'B'C'D'E'F', inscribed in the Circle (O, R), are equal, then area of ABCDEF = πr^2 = area of A'B'C'D'E'F' (1)

Because circles (O, r) & (O, R) are concentric, the 12 equilateral triangles Oab, Obc, Ocd, Ode, Oef, Ofg, Ogh, Ohi, Oij, Ojk, Okl & Ola are equal. This shows that 12 chords of the circle (O, r) are equal,

ab = bc = cd = de = ef = fg = gh = hi = ij = jk = kl = la (2)

By (1) & (2), the inscribed dodecagon abcdefghijkl in the resulting circle (O, r) is regular as required.

From this regularity result, the radius r of the resulting circle (O, r) is determined by the distance from any intersection point (e.g., a) to O. This satisfies the area equivalence condition

πr^2 (O, r) = A of ABCDEF = A of A'B'C'D'E'F',
completing the construction.]

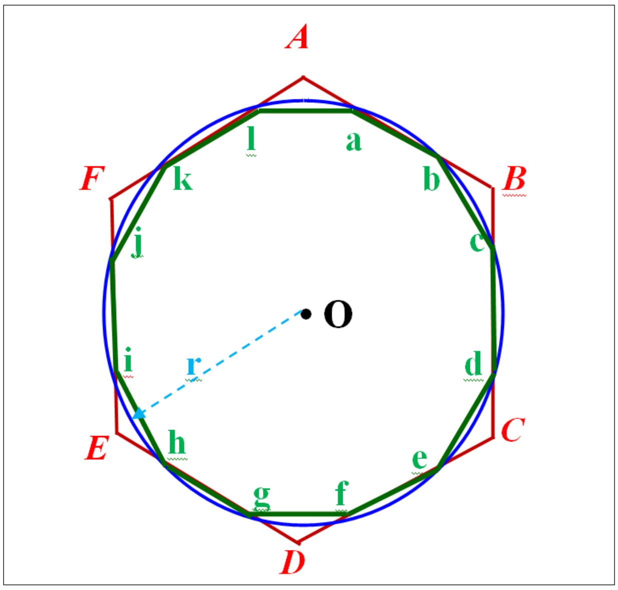
It is demonstrated that the construction of a circle (O, r) with area equal to a regular hexagon ABCDEF can be derived geometrically. The intersection points of (O, r) with ABCDEF and its symmetric counterpart A′B′C′D′E′F′ form a regular dodecagon, validating the relationship between the circle's radius r and the hexagon's geometry. This proof adheres strictly to the geometric principles outlined in the original text and references the provided figures.

The **Key Geometric Relationships** is illustrated as follow:

Hexagon ABCDEF ⇄ *Circle (O, R) and 12 points a, b, c, d, e, f, g, h, i, j, k & l*

Resulting Circle (O, r) ⇄ *Hexagon A'B'C'D'E'F'*

Moreover, the regular dodecagon abcdefghijkl is also an indicator to construct the resulting circle (O, r), as illustrated in the following figure:



The above indicator can be defined as a "**Circling Ruler**" for a **regular hexagon**.

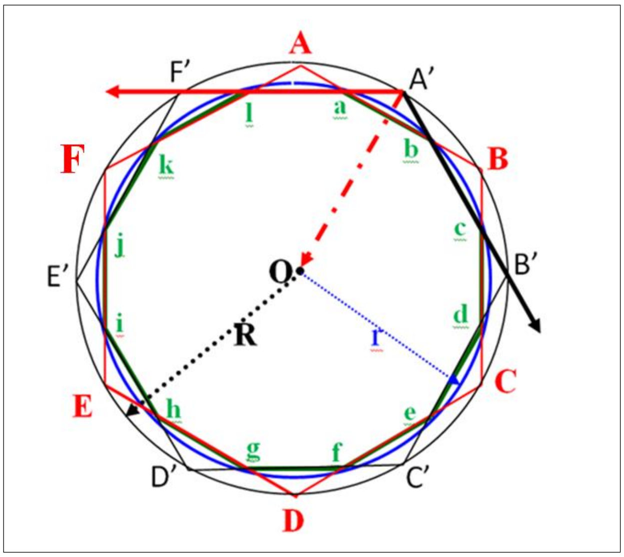


Figure 2: ***Shape* abcdefghhijkl *is the regular dodecagon, inscribed in the resulting circle* (O, r). *The red shape* ABCDEF *is the given regular hexagon to be circled.***

I.3 DEFINITION:

Given a regular hexagon, area A, then any regular dodecagon formed by the hexagon and its symmetric counterpart shape is defined as a "**Circling Ruler**" for the **given hexagon** to be circled (Figure 3 below).

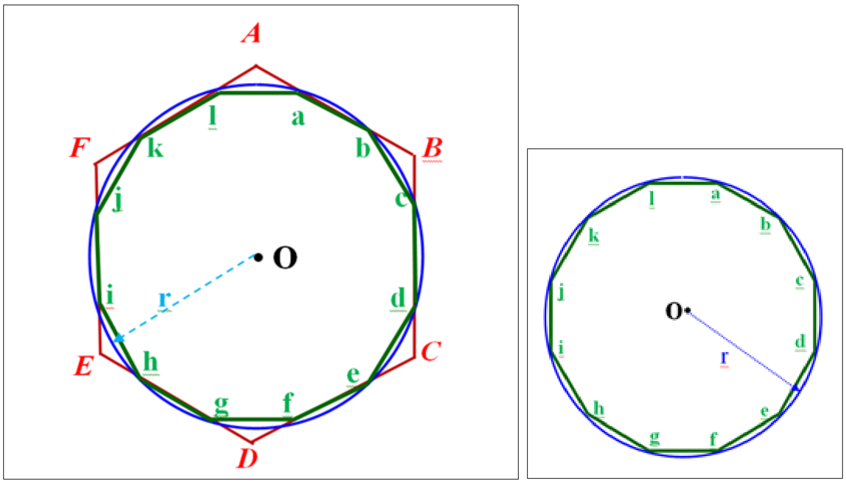


Figure 3: The "Circling Ruler" abcdefghijkl ***(green colour) of a given hexagon ABCDEF is the regular dodecagon inscribed in both the given hexagon and the resulting Circle (O, r).***

Part II: CIRCLING A REGULAR HEXAGON WITH STRAIGHTEDGE AND COMPASS

CORE THEOREM

Given a regular hexagon ABCDEF then its CIRCLING RULER is a mathematics tool to construct a circle (O, r) that has the same area as the area of the given hexagon, with a straightedge & compass in Euclidean Geometry.

PROOF:

Let ABCDEF be a regular hexagon with area A, then by Theorem 2, ABCDEF and its symmetric counterpart A′B′C′D′E′F′ form a regular dodecagon **abcdefghijkl.** Also by Theorem 2 (Section I.2, Part I, above), this dodecagon is an inscribed regular shape in a circle (O, r) where radius r is given by line segment **Oa** of the dodecagon for constructing this circle with area πr^2 = area A of the given hexagon. Because (O, r) is constructed by straightedge & compass with the line segment **Oa** of the dodecagon **a b c d e f g h I j k l,** it is the Circling Ruler as required and described in Figure 4, below.

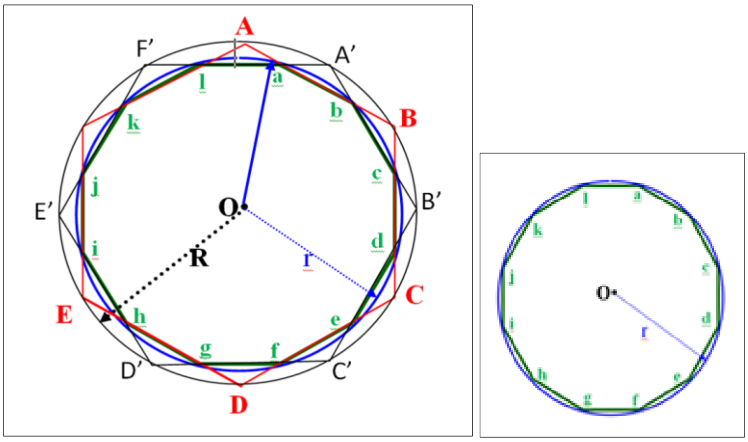


Figure 4: ***The given regular hexagon, the circumscribed circle (O, R), the symmetric counterpart A'B'C'D'E'F', the Circling Ruler*** **abcdefghijkl** ***and the resulting circle (O, r).***

Part IV

DISCUSSION AND CONCLUSION

This study addresses the classical question of whether mathematicians can use only a compass and straightedge to construct a circle with the same area as a given regular hexagon. Through independent research, an exact solution to "*Circling the Square*" problem is derived, demonstrating that if one can construct a circle for a square, it is equally feasible for a regular hexagon [7].

The present paper establishes the certainty of constructing a circle (O, r) with an area πr^2 that is precisely equal to the area A of the given regular hexagon. The construction method employed in this work diverges from approximation techniques, relying solely on Euclidean geometry and the traditional tools of a straightedge and compass.

The motivation for this research stems from the solution to the "*Circling the Square*" problem, published by Scholars Academic and Scientific Publishers (on SJPMS) on 02/05/2024 [7] (DOI : 10.36347/sjpms.2024.v11i05.001,

https://saspublishers.com/media/articles/SJPMS_115_54-64.pdf). The success in circling a square using only a straightedge and compass prompted the question: *if such a construction is possible for a square, why does it seem more challenging to apply the same method to a regular hexagon?* In the context of this study, the given regular hexagon ABCDEF is shown to be circumscribed by a circle (O, R), which also contains a symmetric counterpart A′B′C′D′E′F′. These two hexagons together form a regular dodecagon, inscribed within both hexagons and concentric with them.

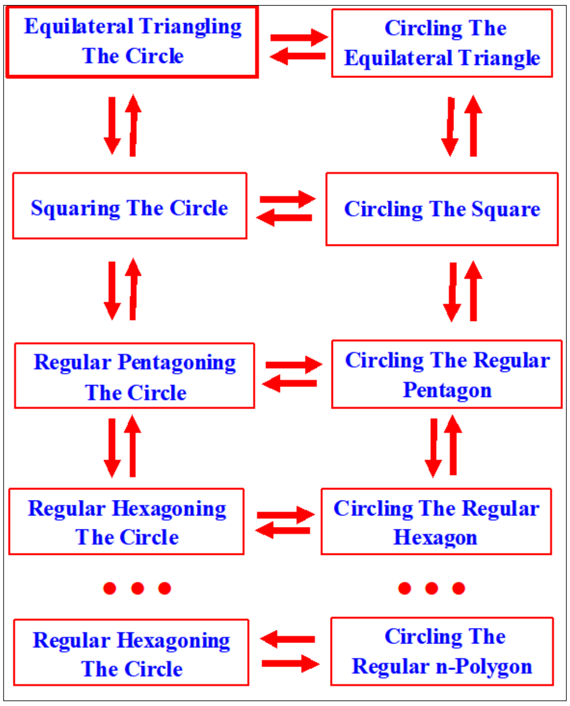
The regular dodecagon becomes the cornerstone of the solution for constructing the required circle (O, r) with an area equal to that of the given hexagon. The radius r of the desired circle is precisely determined as the distance from the centre O to any of the dodecagon's vertices. This construction thus offers an exact solution to the problem of "Circling A Regular Hexagon" within the framework of Euclidean geometry, using only a straightedge and compass. Furthermore, the area of the resulting circle (O, r) is rigorously proven to be equal to the area of the original hexagon, as required.

This paper also introduces a novel mathematical tool, referred to as the "Circling Ruler", which facilitates the construction of a circle with an area equal to that of a given regular hexagon. The paper further demonstrates the following key results:

1. The concentric resulting circle intersects the given regular hexagon at the twelve vertices, forming a regular dodecagon. The centre of the circle and one vertex of the dodecagon determine the required radius of the circle (O, r).
2. The resulting construction inscribes a regular dodecagon within the resulting circle (O, r), confirming the geometric relationship.

Future research could extend this framework to new challenges, such as constructing a circle with a given area for other regular polygons, such as the heptagon. Specifically, an open problem remains: how to construct a circle, using only a straightedge and compass, with an area equal to that of a *regular heptagon* with given side length a and area A, without relying on the traditional constant π.

Additionally, this study opens the door to new investigations into the "*Regular Heptagoning A Circle*" problem within Euclidean geometry, once again using only a straightedge and compass. The results of this research may also contribute to further explorations stemming from the ancient "Squaring the Circle" problem, as depicted in the following diagram:



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REFERENCES

1. Karl Popper, 21 February 2002, The Logic of Scientific Discovery, 2nd edition, page 544, London, Routledge, ISBN9780203994627, DOI:https://doi.org/10.4324/9780203994627, [Publisher link].
2. Rolf Wallisser, On Lambert's proof of the irrationality of π, Published by De Gruyter 2000, https://www.semanticscholar.org/paper/On-Lambert's-proof-of-the-irrationality-of-%CF%80-Wallisser/43f1fe3182f2233a1a9f313649547d13ea7311c7.
3. Laczkovich, M., (1997), "On Lambert's proof of the irrationality of π", The American Mathematical Monthly. 104 (5): 439–443, doi:10.1080/00029890.1997.11990661, JSTOR 2974737, MR 1447977.
4. Wantzel, L., (1837), "Recherches sur les moyens de reconnaître si un problème de géométrie peut se résoudre avec la règle et le compas" [Investigations into means of knowing if a problem of geometry can be solved with a straightedge and compass], Journal de Mathématiques Pures et Appliquées (in French). 2: 366–372.
5. T. Albertini, La quadrature du cercle d'ibn al-Haytham: solution philosophique ou mathématique?, *J. Hist. Arabic Sci.* 9 (1-2) (1991), 5-21, 132. https://www.academia.edu/9342794/_La_quadrature_du_cercle_dIbn_al_Haytham_solution_math%C3%A9matique_ou_philosophique_Ibn_al_Haythams_Quadratura_Circuli_a_mathematical_or_a_philosophical_solution_translation_and_commentary_Journal_for_the_History_of_Arabic_Science_9_1991_5_21.
6. Tran Dinh Son, 31/10/2024, "Exact Squaring the Circle with Straightedge and Compass Only", Scholars Journal of Physics, Mathematics and Statistics | Volume-11 | Issue-10, DOI: https://doi.org/10.36347/sjpms.2024.v11i10.004, https://www.academia.edu/125965397/Scholars_Journal_of_Physics_Mathematics_and_Statistics, https://saspublishers.com/article/20927/.
7. Tran Dinh Son, 2024, "Circling the Square with Straightedge & Compass in Euclidean Geometry" by *Tran Dinh Son, in* Sch J Phys Math Stat | 54-64 DOI: 10.36347/sjpms.2024.v11i05.001, https://saspublishers.com/journal-details/sjpms/149/1453/.
8. Tran Dinh Son, 22 May 2023, "Exact Angle Trisection with Straightedge and Compass by Secondary Geometry", *IJMTT 22 May 2023,* https://doi.org/10.14445/22315373/IJMTT-V69I5P502
   Exact Angle Trisection with Straightedge and Compass by Secondary Geometry (ijmttjournal.org) https://www.academia.edu/103490898/Exact_Angle_Trisection_with_Straightedge_and_Compass_by_Secondary_Geometry
   https://www.academia.edu/ai_review/115747657.
9. Tran Dinh Son, 13/02/2025, "Exact Doubling the Cube with Straightedge and Compass by Euclidean Geometry", *IJMTT 29 August 2023,* Exact Doubling The Cube with Straightedge and Compass by Euclidean Geometry (ijmttjournal.org)
10. https://doi.org/10.14445/22315373/IJMTT-V69I8P506
    https://www.semanticscholar.org/me/library/all, Sch J Phys Math Stat | 24-32, DOI: https://doi.org/10.36347/sjpms.2025.v12i02.001, *SJPMS 13/02/2025,* https://doi.org/10.36347/sjpms.2025.v12i02.001 file:///C:/Users/buung_000/Documents/Downloads/SJPMS_122_24-32%20(3).pdf.
11. Tran Dinh Son, 22/08/2024, Circling a Regular Pentagon with Straightedge and Compass in Euclidean

Geometry,

1. Scholars Journal of Physics, Mathematics and Statistics, Sch J Phys Math Stat ISSN 2393-8056 (Print) | ISSN 2393-8064,
2. DOI: https://doi.org/10.36347/sjpms.2024.v11i08.001 ,
3. https://saspublishers.com/media/articles/SJPMS_118_83-88_FT.pdf.
4. Tran Dinh Son, 16/09/2024, Pentagoning the Circle with Straightedge & Compass, Sch J Phys Math Stat ISSN 2393-8056 (Print) | ISSN 2393-8064 (Online), DOI: https://doi.org/10.36347/sjpms.2024.v11i09.001 ,
5. https://saspublishers.com/media/articles/SJPMS_119_101-107.pdf.
6. Tran Dinh Son, 15/03/2025, Regular Triangling a Circle with Straightedge and Compass in Euclidean Geometry, Scholars Journal Of Physics Mathematics And Statistics (SJPMS), DOI: https://doi.org/10.36347/sjpms.2025.v12i03.003
7. https://saspublishers.com/media/articles/SJPMS_123_59-65.pdf,
8. https://saspublishers.com/article/21784/
9. Tran Dinh Son, 16/06/2025, Circling an Equilateral Triangle with Straightedge and Compass in Euclidean Geometry, Scholars Journal of Physics, Mathematics and Statistics | Volume-12 | Issue-05, Published: June 16, 2025 | 20 22
10. DOI: https://doi.org/10.36347/sjpms.2025.v12i05.001, Pages: 140-148, Archive

link- https://saspublishers.com/journal-details/sjpms/166/1621/, Article link - https://saspublishers.com/article/22406/.
11. Peter Malcolm Johnson, January 31, 2025, An analysis of Euclid's geometrical foundations, (arXiv:2501.17406v2 [math.MG] 30 Jan 2025), https://www.semanticscholar.org/reader/b3fcda47c55e12621d0f4805c94bb52e902e54d0.
12. Frédéric Beatrix, 2022, Squaring the circle like a medieval master mason, Parabola Volume 58, Issue 2 (2022), https://www.parabola.unsw.edu.au/sites/default/files/2024-02/vol58_no2_2.pdf, vol58_no2_2.pdf .
13. S. Ramanujan, 1913, Squaring the circle, Journal of Indian Mathematical Society 5 (1913), 132, Published in the *Journal of the Indian Mathematical Society*, v, 1913, page 132, https://en.wikisource.org/wiki/Squaring_the_circle, ramworks.dvi.
14. H.V. Chu, 16/03/2022, Squaring the Circle Revisited, https://arxiv.org/pdf/1908.01202.pdf, 2019, *last revised 2 Feb 2025 (this version, v2),* arXiv:1908.01202 [math.GM], arXiv:1908.01202v2 [math.GM], Pi Mu Epsilon Journal 15 (2020), 139-143, https://doi.org/10.48550/arXiv.1908.01202, Squaring the Circle Revisited, Hùng Việt Chu hungchu2@illinois.edu Department of Mathematics, University of Illinois Urbana-Champaign, Urbana, IL 61820, USA.
15. Steve Nadis, February 8, 2022, An Ancient Geometry Problem Falls to New Mathematical Techniques, https://www.quantamagazine.org/an-ancient-geometry-problem-falls-to-new-mathematical-techniques-20220208/.